LECTURE NOTES

On

NETWORK THEORY

(BEES2211)

3rd Semester ETC Engineering

Prepared by,

Manjushree Jena

Jemimah Digal

Monalisha Nayak

INDIRA GANDHI INSTITUTE OF TECHNOLOGY,

SARANG

NETWORK THEORY

Ms. Manjushree Jena
Dept:-Electronics and Telecommunication

MODULE -I

NETWORK TOPOLOGY

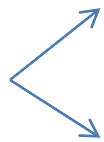
1. **Introduction**: When all the elements in a network are replaces by lines with circles or dots at both ends, configuration is called the graph of the network.

A. Terminology used in network graph:-

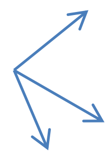
(i) **Path:-** A sequence of branches traversed in going from one node to another is called a path.

(ii) **Node:-** A node point is defined as an end point of a line segment and exits at the junction between two branches or at the end of an isolated branch.

(iii) **Degree of a node:-** It is the no. of branches incident to it.



2-degree



3-degree

(iv) **Tree:-** It is an interconnected open set of branches which include all the nodes of the given graph. In a tree of the graph there can't be any closed loop.

(v) **Tree branch(Twig):-** It is the branch of a tree. It is also named as twig.

(vi) **Tree link(or chord):-** It is the branch of a graph that does not belong to the particular tree.

(vii) **Loop:-** This is the closed contour selected in a graph.

(viii) **Cut-Set:-** It is that set of elements or branches of a graph that separated two parts of a network. If any branch of the cut-set is not removed, the network remains connected. The term cut-set is derived from the property designated by the way by which the network can be divided in to two parts.

(ix) **Tie-Set:-** It is a unique set with respect to a given tree at a connected graph containing on chord and all of the free branches contained in the free path formed between two vertices of the chord.

(x) **Network variables:-** A network consists of passive elements as well as sources of energy . In order to find out the response of the network the through current and voltages across each branch of the network are to be obtained.

(xi) **Directed(or Oriented) graph:-** A graph is said to be directed (or oriented) when all the nodes and branches are numbered or direction assigned to the branches by arrow.

(xii) **Sub graph:-** A graph G_s said to be sub-graph of a graph G if every node of G_s is a node of G and every branch of G_s is also a branch of G.

(xiii) **Connected Graph:-** When at least one path along branches between every pair of a graph exits , it is called a connected graph.

(xiv) **Incidence matrix:-** Any oriented graph can be described completely in a compact matrix form. Here we specify the orientation of each branch in the graph and the nodes at which this branch is incident. This branch is called incident matrix.

When one row is completely deleted from the matrix the remaining matrix is called a reduced incidence matrix.

(xv) **Isomorphism:-** It is the property between two graphs so that both have got same incidence matrix.

B. Relation between twigs and links-

Let N=no. of nodes
L= total no. of links
B= total no. of branches

No. of twigs= N-1

Then, L= B-(N-1)

C. Properties of a Tree-

(i) It consists of all the nodes of the graph.

(ii) If the graph has N nodes, then the tree has (N-1) branch.

(iii) There will be no closed path in a tree.

(iv) There can be many possible different trees for a given graph depending on the no. of nodes and branches.

1. FORMATION OF INCIDENCE MATRIX:-

- This matrix shows which branch is incident to which node.
- Each row of the matrix being representing the corresponding node of the graph.
- Each column corresponds to a branch.
- If a graph contain N- nodes and B branches then the size of the incidence matrix [A] will be NXB.

A. Procedure:-

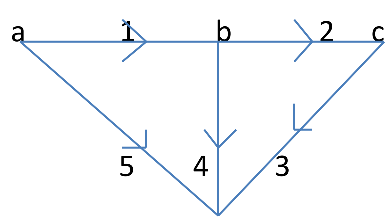
(i) If the branch j is incident at the node I and oriented away from the node, a_{ij}=1. In other words, when a_{ij}=1, branch j leaves away node i.

(ii) If branch j is incident at node j and is oriented towards node i, a_{ij}=-1. In other words j enters node i.

(iii) If branch j is not incident at node i. a_{ij}=0.

The complete set of incidence matrix is called augmented incidence matrix.

<u>Ex-1:-</u> Obtain the incidence matrix of the following graph.



Node-a:- Branches connected are 1& 5 and both are away from the node.
Node-b:- Branches incident at this node are 1,2 &4. Here branch is oriented towards the node whereas branches 2 &4 are directed away from the node.
Node-c:- Branches 2 &3 are incident on this node. Here, branch 2 is oriented towards the node whereas the branch 3 is directed away from the node.
Node-d:- Branch 3,4 &5 are incident on the node. Here all the branches are directed towards the node.

So,

$[A_i]=$

	branch				
Node	1	2	3	4	5
1	1	0	0	0	1
2	-1	1	0	1	0
3	0	-1	1	0	0
4	0	0	-1	-1	-1

B. Properties:-

(i) Algebraic sum of the column entries of an incidence matrix is zero.

(ii) Determinant of the incidence matrix of a closed loop is zero.

C. Reduced Incidence Matrix :-

If any row of a matrix is completely deleted, then the remaining matrix is known as reduced Incidence matrix. For the above example, after deleting row,
we get

$$[A_i']=\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 \end{bmatrix}$$

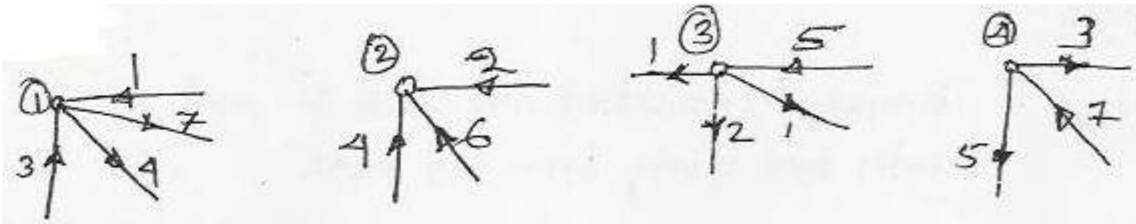
A_i' is the reduced matrix of A_i .

<u>Ex-2</u>: Draw the directed graph for the following incidence matrix.

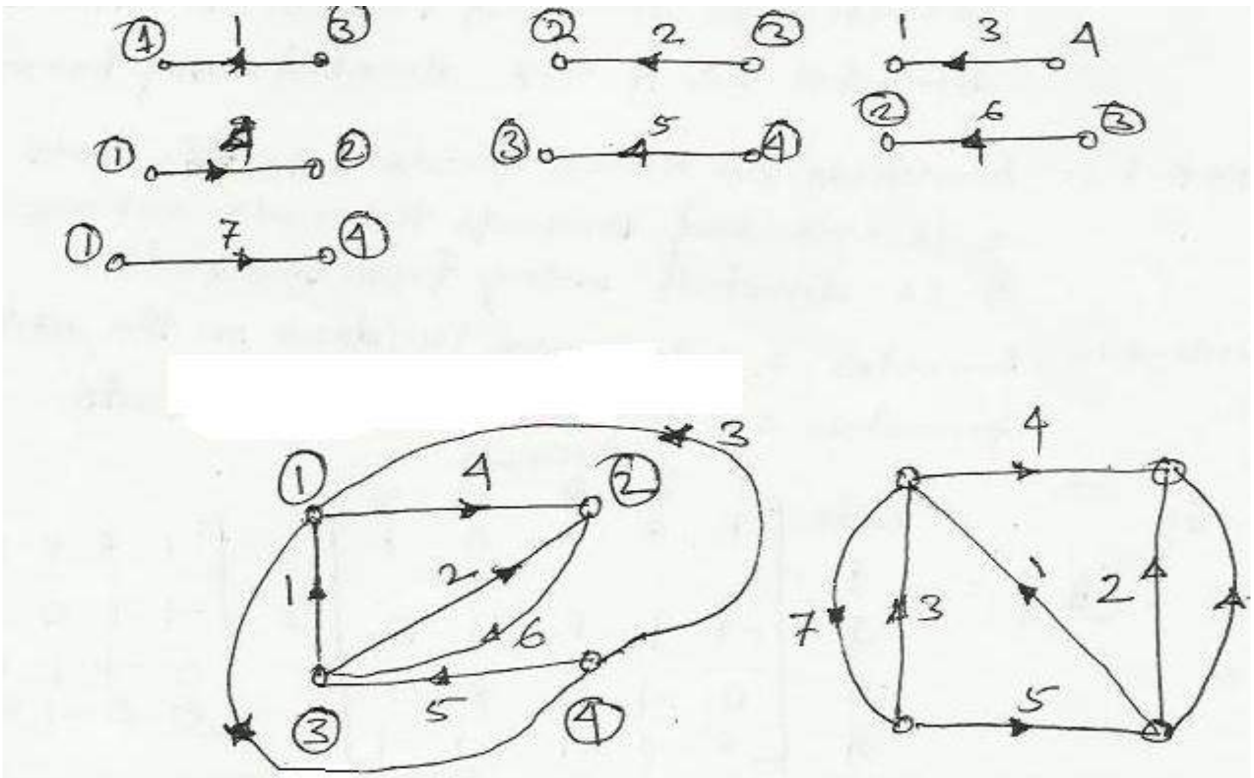
	Branch						
Node	1	2	3	4	5	6	7
1	-1	0	-1	1	0	0	1
2	0	-1	0	-1	0	-1	0
3	1	1	0	0	-1	1	0
4	0	0	1	0	1	0	-1

Solution:-

<u>From node</u>



<u>From branch</u>



<u>Tie-set Matrix:</u>

	Branch 1	2	3	4	5
Loop currents I_1	1	0	0	1	1
I_2	-1	-1	1	0	-1

$$Bi = \begin{vmatrix} 1 & 0 & 0 & 1 & 1 \\ -1 & -1 & 1 & 0 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & -1 & 0 & 1 \end{vmatrix}$$

Let V_1, V_2, V_3, V_4 & V_5 be the voltage of branch 1,2,3,4,5 respectively and j_1, j_2, j_3, j_4, j_5 are current through the branch 1,2,3,4,5 respectively.

So, algebraic sum of branch voltages in a loop is zero.
Now, we can write,

$V_1 + V_4 + V_5 = 0$

$V_1 + V_2 - V_3 + V_5 = 0$

Similarly, $j_1 = I_1 - I_2$ $\quad j_2 = -I_2$ $\quad j_3 = I_2$ $\quad j_4 = I_1$

$j_5 = I_1 - I_2$

Fundamental of cut-set matrix:-

A fundamental cut-set of a graph w.r.t a tree is a cut-set formed by one twig and a set of links. Thus in a graph for each twig of a chosen tree there would be a fundamental cut set.

No. of cut-sets=No. of twigs=N-1.

Procedure of obtaining cut-set matrix:-

(i) Arbitrarily at tree is selected in a graph.
(ii) From fundamental cut-sets with each twig in the graph for the entire tree.
(iii) Assume directions of the cut-sets oriented in the same direction of the concerned twig.
(iv) Fundamental cut-set matrix[Q_{kj}]

Q_{kj}=+1; when branch b_j has the same orientation of the cut-set

Q_{kj}=-1; when branch b_j has the opposite orientation of the cut-set

Q_{kj}=0; when branch b_j is not in the cut-set

Fundamental of Tie-set matrix:-

A fundamental tie-set of a graph w.r.t a tree is a loop formed by only one link associated with other twigs.

No. of fundamental loops=No. of links=B-(N-1)

Procedure of obtaining Tie-set matrix:-

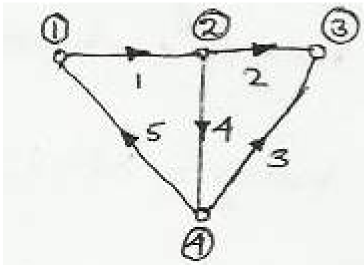
(i) Arbitrarily a tree is selected in the graph.
(ii) From fundamental loops with each link in the graph for the entire tree.
(iii) Assume directions of loop currents oriented in the same direction as that of the link.
(iv) From fundamental tie-set matrix[b_{ij}] where

b_{ij}=1; when branch b_j is in the fundamental loop i and their reference directions are oriented same.

b_{ij}=-1; when branch b_j is in the fundamental loop i but, their reference directions are oriented oppositely.

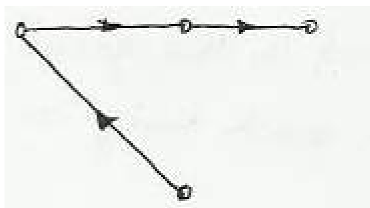
b_{ij}=0; when branch b_j is not in the fundamental loop i .

Ex-3: Determine the tie set matrix of the following graph. Also find the equation of branch current and voltages.



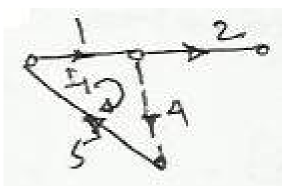
Solution

Tree

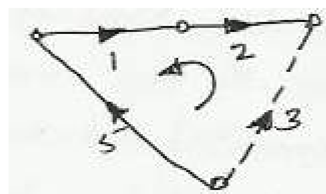


No. of loops= No. of links= 2

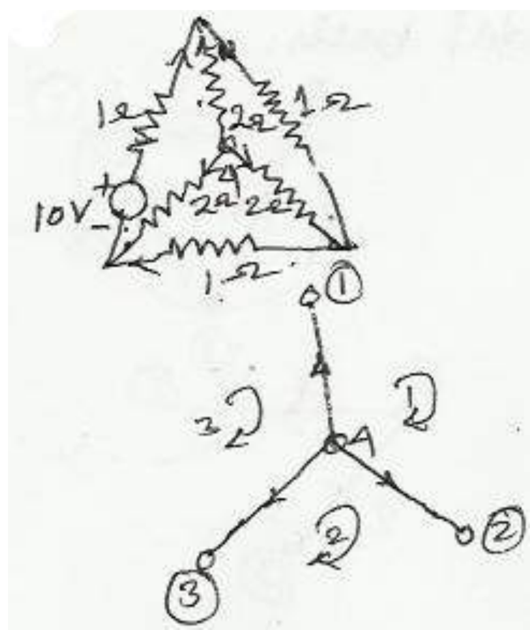
Loop 1



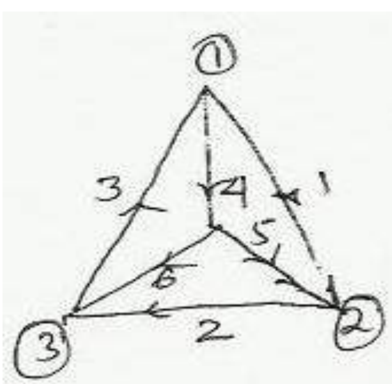
Loop 2



Q1. Draw the graph and write down the tie-set matrix. Obtain the network equilibrium equations in matrix form using KVL.



Solution



Tie-set

	1	2	3	4	5	6
I_1	1	0	0	1	-1	0
I_2	0	1	0	0	1	-1
I_3	0	0	1	-1	0	1

$$V_1 + V_4 - V_5 = 0 \qquad j_1 = I_1$$

$$V_2 + V_5 - V_6 = 0 \qquad j_2 = I_2$$

$$V_3 - V_4 + V_6 = 0 \qquad j_3 = I_3$$

Again, $V_1 = e_2 - e_2$ $\qquad i_4 = I_1 - I_3$

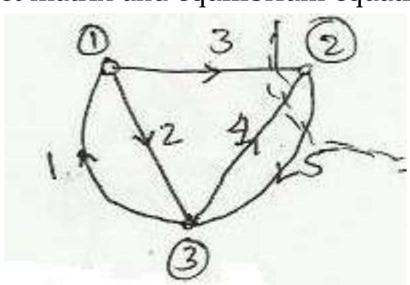
$V_2 = e_3 - e_2$ $\qquad i_5 = I_2 - I_1$

$V_4 = e_4 - e_1$ $\qquad i_6 = I_3 - I_2$

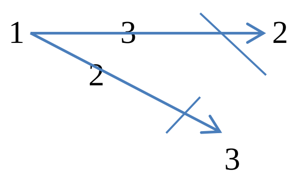
$V_5 = e_2 - e_4$

$V_6 = e_3 - e_4$

Q2. Develop the cut-set matrix and equilibrium equation on nodal basis.



Ans.

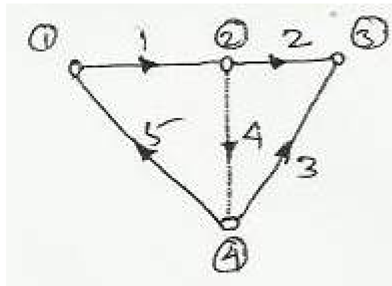


Cut set	1	2	3	4	5
C1	0	0	1	1	-1
C2	-1	1	0	-1	1

$$i_3 + i_4 - i_5 = 0$$

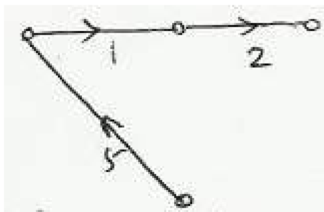
$$-i_1 + i_2 - i_4 + i_5 = 0$$

Ex- Determine the cut-set matrix and the current balance equation of the following graph?

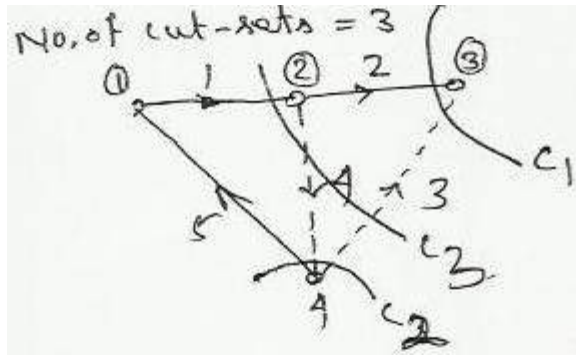


Solution:

Tree



No of twigs=1, 2, 5



Cut-set matrix

	branch				
cut-set	1	2	3	4	5
C1	0	1	1	0	0
C2	0	0	1	-1	1
C3	1	0	1	-1	0

$i_2 + i_3 = 0$

$i_3 - i_4 + i_5 = 0$ where, i_1, i_2, i_3, i_4, i_5 are respective branch currents.

$i_1 + i_3 - i_4 = 0$

Nodal & Mesh Analysis Of Electric Circuits

Node – It is a equipotential point at which two or more circuit elements are joined.

Junction – It is that point of a network where three or more circuit elements are joined.

Branch – It is a part of a network which lies between junction points.

Nodal Analysis

In the nodal analysis it is essential to compute all branch current.

In this method, the number of independent node pair equations needed is one less than the number of junctions in the network.

i.e $\boxed{n = j - 1}$

where n → denotes the no. of independent node equations
j → the no. of junction.

Mesh Analysis

In the mesh analysis KVL is applied around each closed loop & by solving these loop equations, the branch current is determined.

For this method the no. of independent mesh equations needed is

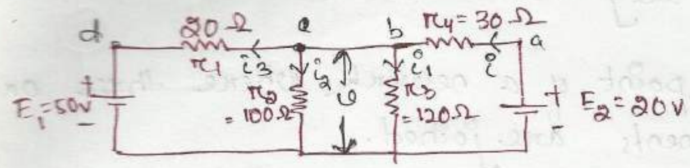
$$\boxed{m = b - (j - 1)}$$

where b → the no. of branches.

Note

If m<n, the mesh method offers advantages while for m>n, the nodal method is preferred.

Ex



Using Nodal method, find the current through r_2.

Sol^n

As node b & c are electrically same, the KCL equation across node b is

$$i = i_1 + i_2 + i_3$$

$$\Rightarrow \frac{20-v}{30} = \frac{v}{120} + \frac{v}{100} + \frac{v-50}{20}$$

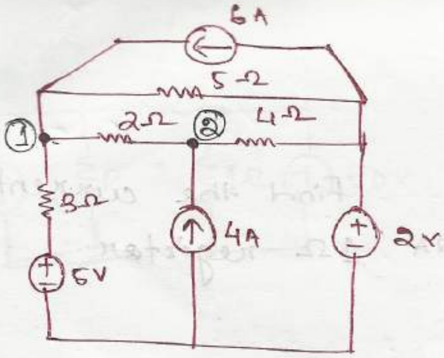
$$\Rightarrow \frac{20}{30} + \frac{50}{20} = v\left[\frac{1}{30} + \frac{1}{120} + \frac{1}{100} + \frac{1}{20}\right]$$

$$\Rightarrow \boxed{v = 31.18\ V}$$

$$i_2 = \frac{v}{100} = \frac{31.18}{100}$$

$$\Rightarrow \boxed{i_2 = .3118\ A = 311.8\ mA}$$

Ex-2



Using Nodal method find the current through the resistors in the circuit.

Sol:

For node '1'

$$\frac{V_1-5}{3}+\frac{V_1-V_2}{2}+\frac{V_1-2}{5}=6$$

$$= V_1\left(\frac{1}{3}+\frac{1}{2}+\frac{1}{5}\right)-\frac{V_2}{2} = 6+\frac{2}{5}+\frac{5}{3}$$

$$\Rightarrow \frac{31}{30}V_1-\frac{V_2}{2}-\frac{121}{15}=0 \quad \text{——— (i)}$$

For node '2'

$$\frac{V_2-V_1}{2}+\frac{V_2-2}{4}=4$$

$$= V_2\left(\frac{1}{2}+\frac{1}{4}\right)-\frac{V_1}{2} = 4+\frac{1}{2}$$

$$= \frac{3}{4}V_2-\frac{V_1}{2}-\frac{9}{2}=0 \quad \text{——— (ii)}$$

Solving eqn (i) & (ii) $V_1 = 15.76V$ & $V_2 = 16.51V$

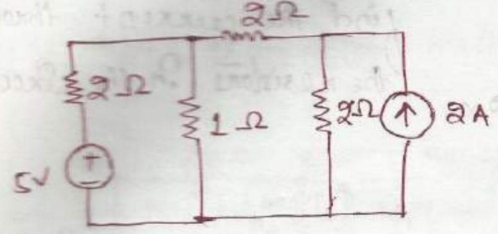
Current through resistor $3\Omega = \frac{V_1-5}{3} = \frac{15.76-5}{3} \simeq 3.6A$

Current through resistor $2\Omega = \frac{V_1-V_2}{2} = \frac{15.76-16.51}{2}$
$= -0.375A$

Current through $5\Omega = \frac{V_1-2}{5} = \frac{15.76-2}{5} = 2.76A$

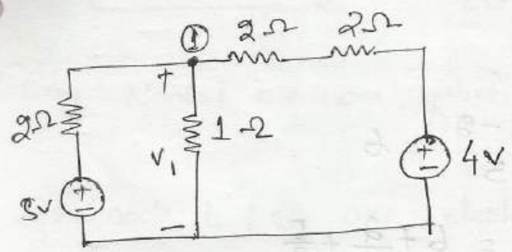
Current through $4\Omega = \frac{V_2-2}{4} = \frac{16.51-2}{4} = 3.63A$

Ex-3



Find the current in 1Ω register.

Sol



Using Nodal analysis

$$\frac{V_1-5}{2}+\frac{V_1-4}{4}+\frac{V_1}{1}=0$$

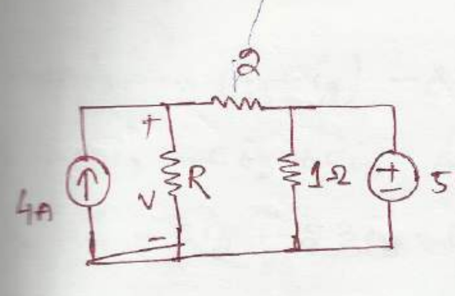
$$\Rightarrow V_1\left(\frac{1}{2}+\frac{1}{4}+1\right)=\frac{5}{2}+1+1$$

$$\Rightarrow V_1=2V$$

Current across 1Ω $=\frac{V_1}{1}=\frac{2}{1}=2V$

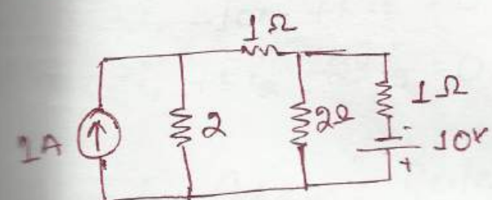
Q-1

Find current through through the resistor r_2 by nodal method.

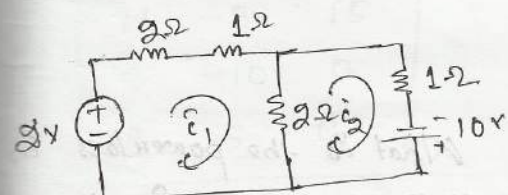


What is the value of V such that the power supplied by both the sources are equal to each other.

Ex-4



Using Mesh analysis, obtain the current through the 10V battery for the circuit.



Applying KVL in loop-1

$$2 - 3i_1 - 2(i_1 - i_2) = 0$$

$$\Rightarrow 2 - 3i_1 - 2i_1 + 2i_2 = 0$$

$$\Rightarrow 2 - 5i_1 + 2i_2 = 0$$

$$\Rightarrow 5i_1 - 2i_2 = 2 \quad \text{——— (1)}$$

Applying KCL in loop-2

$$10 - 2(i_2 - i_1) - i_2 = 0$$

$$\Rightarrow 10 - 3i_2 + 2i_1 = 0$$

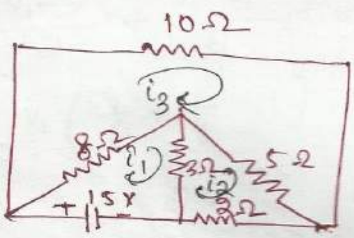
$$\Rightarrow 2i_1 - 3i_2 = -10 \quad \text{——— (2)}$$

$$2(5i_1 - 2i_2 = 2)$$
$$5(2i_1 - 3i_2 = -10)$$

$$11 i_2 = 49$$
$$\Rightarrow \boxed{i_2 = 4.91A}$$

$$5i_1 - 9.82 = 2$$
$$\Rightarrow 5i_1 = 11.82$$
$$\Rightarrow \boxed{i_1 = 2.36A}$$

Ex - 5



What is the power loss in the 10 Ω resistor in the network using mesh method.

sol^n

$$15 - 8(i_1 - i_3) - 3(i_1 - i_2) = 0$$
$$\Rightarrow 15 - 8i_1 + 8i_3 - 3i_1 + 3i_2 = 0$$
$$\Rightarrow 11i_1 - 3i_2 - 8i_3 = 15 \quad \text{——— (1)}$$
$$-3(i_2 - i_1) - 5(i_2 - i_3) - 2i_2 = 0$$
$$\Rightarrow -3i_2 + 3i_1 - 5i_2 + 5i_3 - 2i_2 = 0$$
$$\Rightarrow 3i_1 - 10i_2 + 5i_3 = 0 \quad \text{——— (2)}$$

$$-10i_3 - 5(i_3 - i_2) - 8(i_3 - i_1) = 0$$

$$\Rightarrow -10i_3 - 5i_3 + 5i_2 - 8i_3 + 8i_1 = 0$$

$$\Rightarrow 8i_1 + 5i_2 - 23i_3 = 0 \quad \text{——} (3)$$

So, the loop eq^ns are

$$11i_1 - 3i_2 - 8i_3 = 15$$

$$3i_1 - 10i_2 + 5i_3 = 0$$

$$8i_1 + 5i_2 - 23i_3 = 0$$

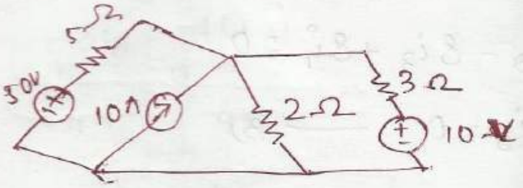
Solving by Cramer's Rule, i_3 current is

$$i_3 = \frac{\begin{vmatrix} 11 & -3 & 15 \\ 3 & -10 & 0 \\ 8 & 5 & 0 \end{vmatrix}}{\begin{vmatrix} 11 & -3 & -8 \\ 3 & -10 & 5 \\ 8 & 5 & -23 \end{vmatrix}} \simeq 1.234$$

Power loss in 10Ω resistor is $= i_3^2 \times 10$

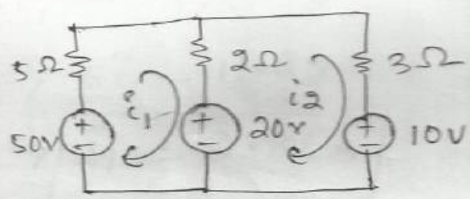
$$= (1.23)^2 \times 10 = 15.13\,W.$$

Ex-6



Using mesh analysis, find the current flow through 50V source in the network.

Sol^n



$50 - 5i_1 - 2(i_1 - i_2) - 20 = 0$

$\Rightarrow 30 - 5i_1 - 2i_1 + 2i_2 = 0$

$\Rightarrow 7i_1 - 2i_2 = 30 \quad \text{—— (1)}$

$20 - 2(i_2 - i_1) - 3i_2 - 10 = 0$

$\Rightarrow 10 - 2i_2 + 2i_1 - 3i_2 = 0$

$\Rightarrow 2i_1 - 5i_2 = -10 \quad \text{—— (2)}$

$5(7i_1 - 2i_2 = 30)$

$2(2i_1 - 5i_2 = -10)$

$31 i_1 = 170$

$\Rightarrow \boxed{i_1 = 5.48\,A}$

Network Theorem

1) Thevenin's Theorem

Any two terminal bilateral linear d.c circuit can be replaced by an equivalent circuit consisting of a voltage source & a series resistor.

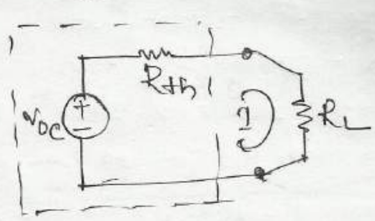
Steps

(i) Remove the load resistor (R_L) & find the open circuit voltage (V_{oc}) across the open circuited load terminal.

(ii) Deactivate the constant sources (for voltage source, remove it by internal resistance & for current source delete the source by open circuit) & find the internal resistance looking through the open circuited load terminal. Let this resistance be R_{Th}.

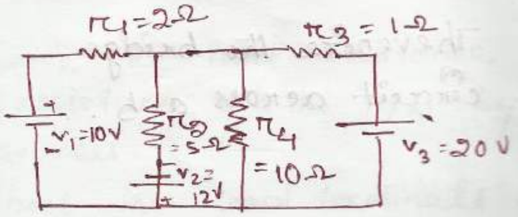
(iii) Obtain Thevenin's equivalent circuit by placing R_{Th} in series with V_{oc}.

(iv) Reconnect R_L across the load terminals.



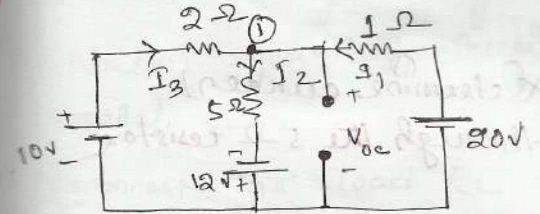
The load current $(I) = \frac{V_{oc}}{R_{Th}+R_L}$

Ex-1



Find the current through 10 Ω resistor using Thevenin's theorem.

Sol^n



At node 1

$$I_1 + I_3 - I_2 = 0$$

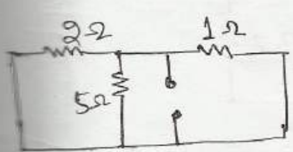
$$\Rightarrow \frac{20-V_1}{1} + \frac{10-V_1}{2} - \frac{V_1+12}{5} = 0$$

$$\Rightarrow 20 + 5 - \frac{12}{5} = V_1 + \frac{V_1}{2} + \frac{V_1}{5}$$

$$\Rightarrow 22.6 = 1.7 V_1$$

$$\Rightarrow \boxed{V_1 = 13.29 V}$$

$$\boxed{V_1 = V_{oc} = 13.29 V}$$

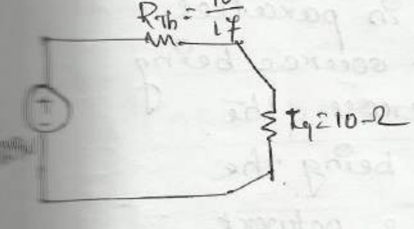


$$R_{Th} = 2 \parallel 5 \parallel 1$$

$$= \frac{2 \times 5}{7} \parallel 1 = \frac{10}{7} \parallel 1$$

$$= \frac{10/7}{\frac{10}{7}+1} = \frac{10/7}{17/7}$$

$$\Rightarrow \boxed{R_{Th} = \frac{10}{17} \Omega}$$

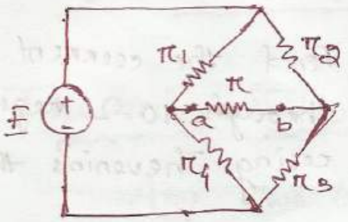


$$I = \frac{V_{oc}}{R_{Th} + r_4}$$

$$= \frac{13.29}{\frac{10}{17} + 10}$$

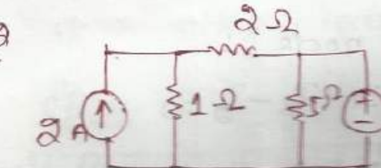
$$\Rightarrow \boxed{I = 1.26 A}$$

Q-1



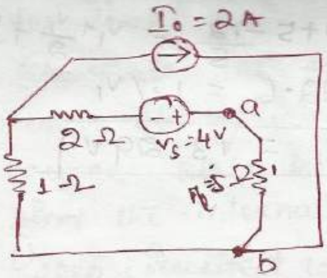
Theveneize the bridge circuit across a-b.

Q-2



determine current through the 5 Ω resistor.

Q-3



Find the powerloss in r_L using Thevenin's theorem.

Norton's Theorem

A linear active network consisting of independent and dependent voltage & current sources & linear bilateral network elements can be replaced by an equivalent circuit consisting of a current source in parallel with a resistance, the current source being the short circuited current across the load terminal & the resistance being the internal resistance of the source network looking through the open circuited load terminals.

Steps

(i) Remove the load resistance & find the internal resistance of the N/W by deactivating the constant sources.

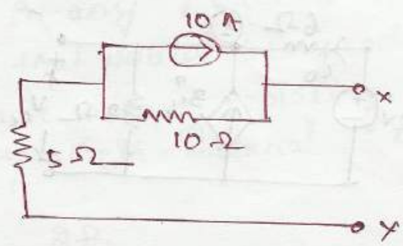
(ii) Short the load terminals & find the short circuit current flowing through the load terminal.

(iii) Norton's equivalent circuit is drawn by keeping R_{int} parallel to i_{sc}.

(iv) Reconnect the load R_L across the load terminal & the current through it is,

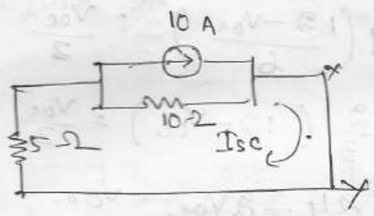
$$\boxed{I_L = i_{s.c}\frac{R_{int}}{R_{int}+R_L}}$$

Ex-1



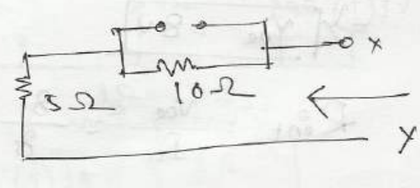
Find Norton's equivalent circuit & current

Sol^n

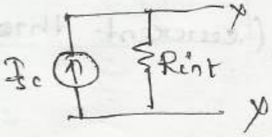


$$I_{sc} = \frac{10\ \ 2}{15\ \ 3} \times 10$$

$$\boxed{I_{sc} = 6.67\,A}$$

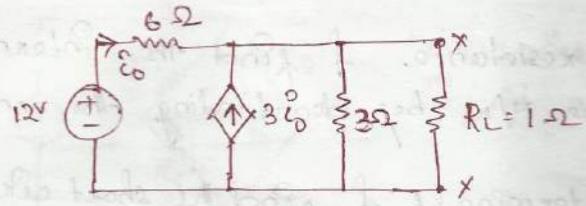


$$\boxed{R_{int} = 5+10 = 15\,\Omega}$$



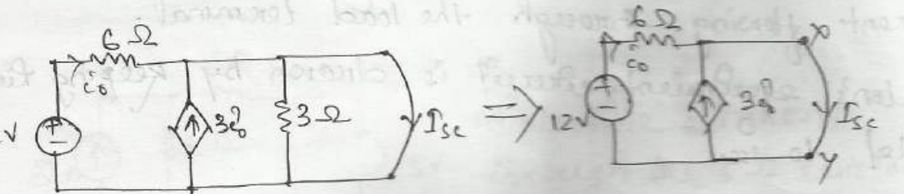
$$\boxed{I_N = I_{sc} = 6.67\,A}$$

Ex-2



Find current through R_L.

Sol^n

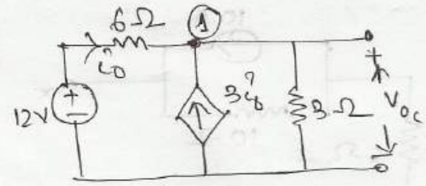


$$I_{sc} = i_0 + 3i_0 = 4i_0$$

$$i_0 = \frac{12}{6} = 2A$$

So, $I_{sc} = 4 \times 2 = 8A$

At node-1



$$i_0 + 3i_0 = \frac{V_{oc}}{3}$$

$$\Rightarrow 4i_0 = \frac{V_{oc}}{3}$$

$$\Rightarrow 4\left(\frac{12 - V_{oc}}{6}\right) = \frac{V_{oc}}{3}$$

$$\Rightarrow \frac{2}{3}(12 - V_{oc}) = \frac{V_{oc}}{3}$$

$$\Rightarrow 24 - 2V_{oc} = V_{oc}$$

$$\Rightarrow \boxed{V_{oc} = 8V}$$

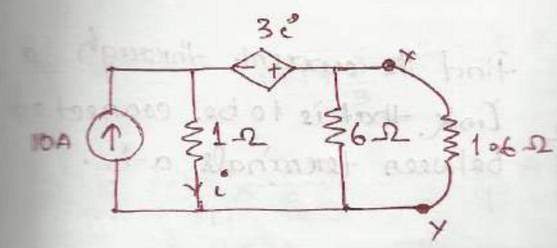
$$R_{int} = \frac{V_{oc}}{I_{sc}} = \frac{8}{8} = 1\ \Omega$$

$$I_L \text{ (current through } 1\,\Omega) = I_{sc}\frac{R_{int}}{R_{int} + R_L}$$

$$= \frac{8 \times 1}{1+1} =$$

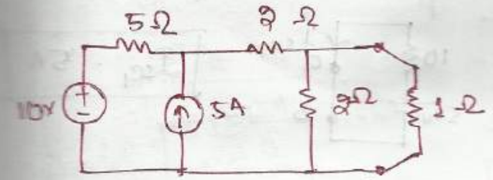
$$\Rightarrow \boxed{I_L = 4A}.$$

Q-1



Find current through 1·6 Ω resistor in the circuit.

Q-2

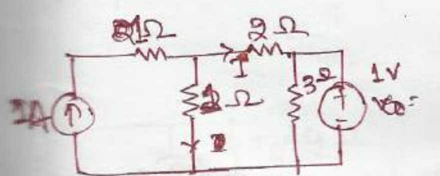


Find powerloss in 1 Ω resistor using Norton's Theorem.

3. Superposition Theorem

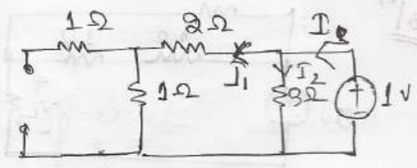
If a number of voltage or current sources are acting simultaneously in a linear network, the resultant current in any branch is the algebric sum of the current that would be produced in it, when each source acts alone replacing all other independent source by their internal resistance.

Ex-1



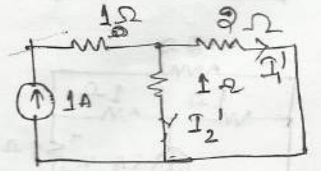
Find I in the circuit

Sol



$$I_s = \frac{1}{3 \| 3} = \frac{1}{1\cdot 5} = \frac{2}{3} A$$

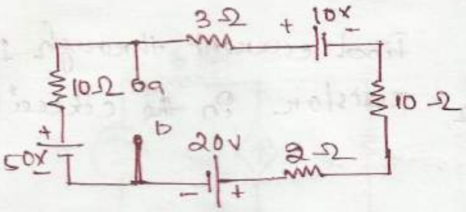
$$I_1 = \frac{1}{3} A$$



$$I_1' = \frac{1}{3} \times 1 = \frac{1}{3}$$

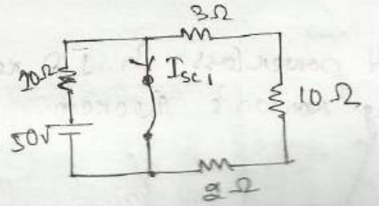
$$I = I_1 - I_1' = \frac{1}{3} - \frac{1}{3} = 0$$

Ex-2.

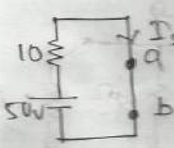


Find the current through a link, that is to be connected between terminals a-b.

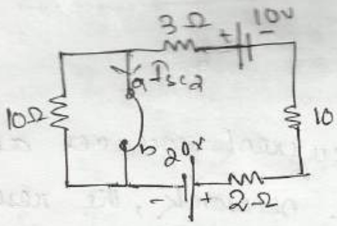
Sol^n



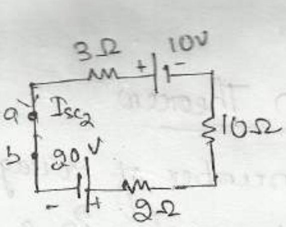
⇒



$I_{sc_1} = 5A$



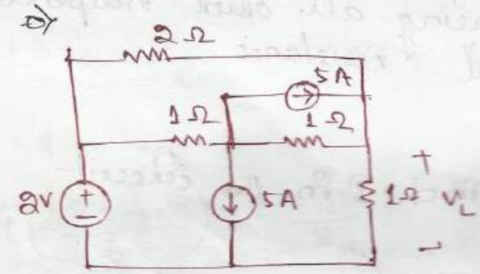
⇒



$I_{sc_2} = \frac{30}{15} = 2A$

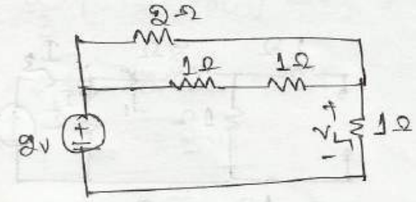
$I_{sc} = I_{sc_1} + I_{sc_2} = 5 + 2 = 7A$

Ex-3

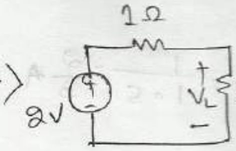


Find V_L.

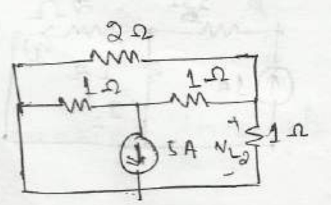
Sol^n



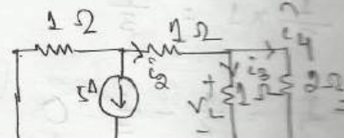
⇒



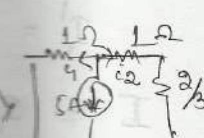
$V_{L_1} = 1V$



⇒



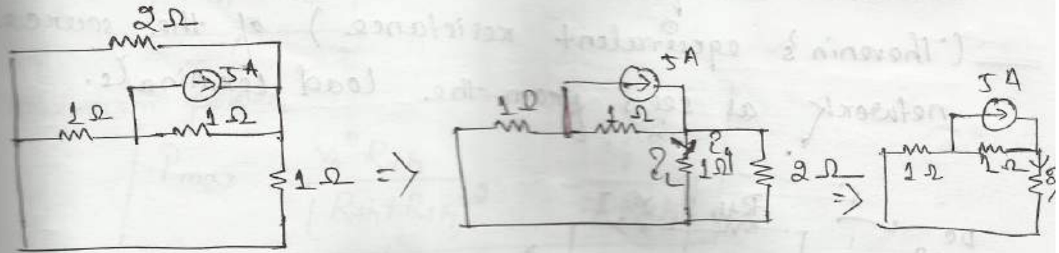
⇒



$i_2 = -5 \times \frac{1}{1+1+\frac{2}{3}} = -5 \times \frac{3}{8} = -\frac{15}{8}$ A

$i_3 = \frac{-15}{8} \times \frac{2}{3} = -\frac{5}{4}$ A

$v_{L_2} = 1 \times -\frac{5}{4} = -\frac{5}{4}$ v



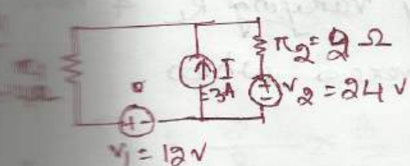
$i_4 = 5 \times \frac{1}{1+\frac{2}{3}} = \frac{15}{5} = 3$ A

$i_L = \frac{2}{3} \times 3 = 2$ A

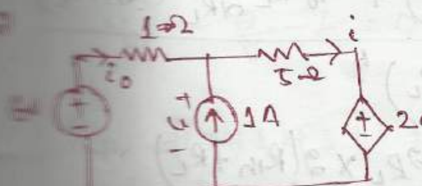
$v_{L_3} = 2 \times 1 = 2$ v

By superposition $v_L = v_{L_1} + v_{L_2} + v_{L_3} = 1 - \frac{5}{4} + 2 = 3 - \frac{5}{4}$

$= \frac{7}{4}$ v



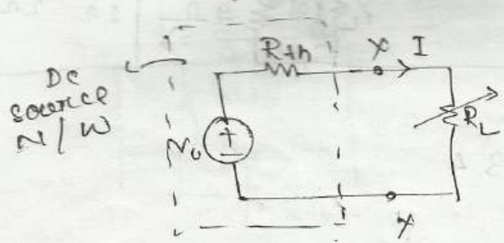
Find current through r_1.



Find i_0 & i.

4. Maximum Power Transfer Theorem

A resistance load, being connected to a dc network, receives maximum power when the load resistance is equal to the internal resistance (Thevenin's equivalent resistance) of the source network as seen from the load terminals.



$$I = \frac{V_0}{R_{th}+R_L}$$

while power delivered to the resistive load is

$$P_L = I^2 R_L = \left(\frac{V_0}{R_{th}+R_L}\right)^2 R_L$$

P_L can be maximised by varying R_L & hence maximum power can be delivered when $(dP_L/dR_L) = 0$

However
$$\frac{dP_L}{dR_L} = \frac{(R_{th}+R_L)^2 \frac{d}{dR_L}(V_0^2 R_L) + V_0^2 R_L \frac{d}{dR_L}(R_{th}+R_L)^2}{(R_{th}+R_L)^4}$$

$$= \frac{(R_{th}+R_L)^2 V_0^2 - V_0^2 R_L \times 2(R_{th}+R_L)}{(R_{th}+R_L)^4}$$

$$= \frac{V_0^2(R_{th}+R_L - 2R_L)}{(R_{th}+R_L)^3} = \frac{V_0^2(R_{th}-R_L)}{(R_{th}+R_L)^3}$$

But $\frac{dP_L}{dR_L} = 0$

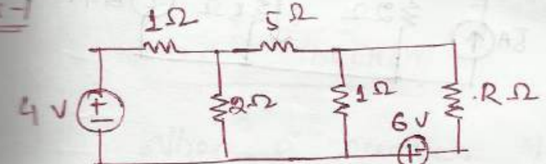
$$\Rightarrow \frac{V_0^2 (R_{th} - R_L)}{(R_{th} + R_L)^2} = 0$$

$$\Rightarrow \boxed{R_{th} = R_L}$$

So, maximum power is

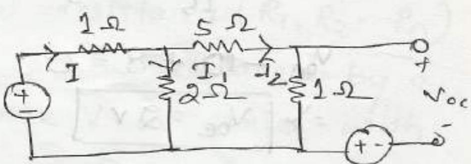
$$\boxed{P_{max} = \frac{V_0^2 R_{th}}{(R_{th} + R_{th})^2} = \frac{V_0^2}{4 R_{th}}}$$

Ex-1



Find the value R in the circuit such that max^m power transfer takes place.

sol^n



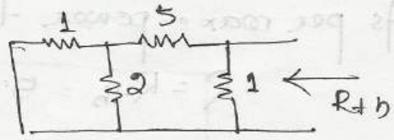
$$I = \frac{4}{1 + (6 \parallel 2)}$$

$$= \frac{4}{1 + \frac{12}{8}} = \frac{32}{20}$$

$$= \frac{8}{5} \text{ A}$$

$$I_2 = \frac{2}{8} \times \frac{8}{5} = \frac{2}{5} \text{ A}$$

$$V_{oc} - \frac{2}{5} - 6 = 0 \Rightarrow V_{oc} = \frac{2}{5} + 6 = 6.4 \text{ V}$$



$$R_{th} = [(2 \parallel 1) + 5] \parallel 1$$

$$= \left(\frac{2}{3} + 5\right) \parallel 1$$

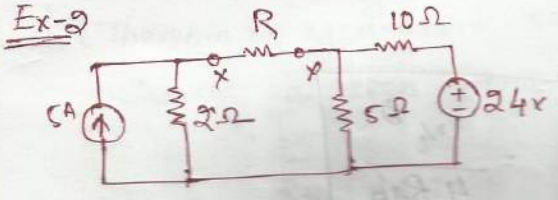
$$= \frac{\frac{17}{3}}{\frac{17}{3} + 1} = \frac{17/3}{20/3} = 17/20 \ \Omega$$

For max. power transfer theorem

$$R = R_{th} = \frac{17}{20}\,\Omega = .85\,\Omega$$

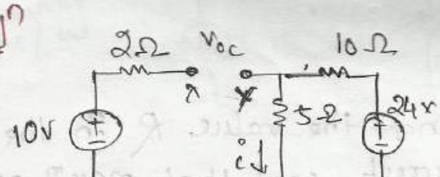
$$P_{max} = \frac{V_0^2}{4R_{th}} = \frac{(6.4)^2}{4 \times .85} = 12W$$

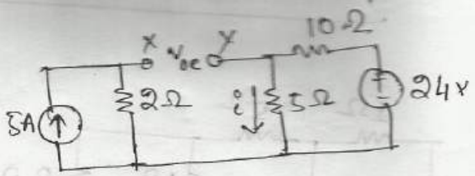
Ex-2



Find the value of R for maximum power transfer.

Soln

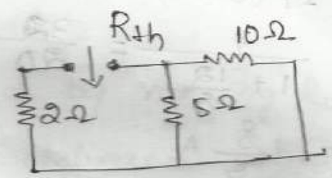




$$i = \frac{24}{15} = \frac{8}{5} = 1.6A$$

$$V_{oc} - 10 + 8 = 0$$

$$\Rightarrow \boxed{V_{oc} = 2v}$$



$$R_{th} = 2 + (10 \| 5)$$

$$= 2 + \frac{50}{15} = 2 + \frac{10}{3}$$

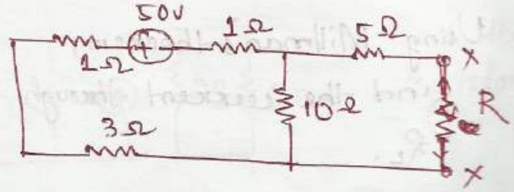
$$= \frac{16}{3} = 5.33\,\Omega$$

As per max. power transfer theorem

$$R = R_{th} = 5.33\,\Omega$$

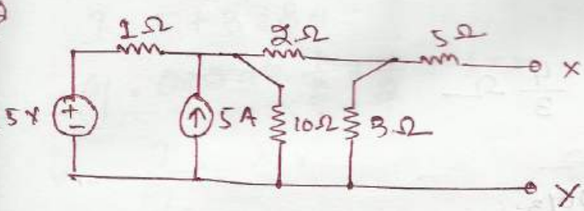
$$P_{max} = \frac{V_{oc}^2}{4R_{th}} = \frac{4}{4 \times 5.33} = \frac{3}{16} = 188\,mW$$

Q-1



find the value of R for max power transfer theorem.

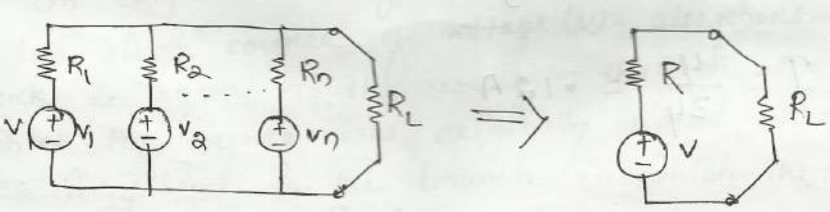
Q-2



What resistance should be connected across x-y in the circuit, such that maxm power is delivered across load resistance.

5. Millman's Theorem

When a number of voltage source ($V_1, V_2 \cdots V_n$) are in parallel having internal resistances ($R_1, R_2 \cdots R_n$) respectively, the arrangement can be replaced by a single equivalent voltage source V in series with an equivalent series resistance R.

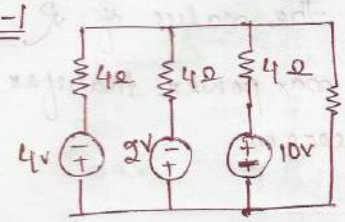


As per Millman's Theorem

$$V = \frac{\pm V_1 G_1 \pm V_2 G_2 \pm \cdots \pm V_n G_n}{G_1 + G_2 + \cdots + G_n}$$

$$R = \frac{1}{G} = \frac{1}{G_1 + G_2 + \cdots + G_n}$$

Ex-1



Using Millman's theorem, find the current through R_L.

Soln

$$R = \frac{1}{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}} = \frac{4}{3}\,\Omega$$

$$V = \frac{-V_1G_1 - V_2G_2 + V_3G_3}{G_1+G_2+G_3}$$

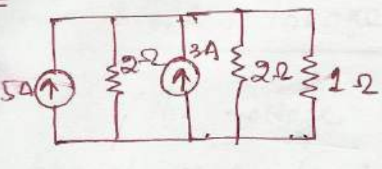
$$= \frac{-4\times\frac{1}{4} - 2\times\frac{1}{4} + 10\times\frac{1}{4}}{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}} = \frac{-1-\frac{1}{2}+\frac{5}{2}}{\frac{3}{4}}$$

$$= \frac{-\frac{3}{2}+\frac{5}{2}}{\frac{3}{4}} = \frac{1}{\frac{3}{4}} = \frac{4}{3}\,V$$

$$I = \frac{V}{R+R_L} = \frac{\frac{4}{3}}{\frac{4}{3}+10} = \frac{4/3}{34/3}$$

$$\Rightarrow I = \frac{4}{34} = .12A$$

Ex-2



Find the current through the 1 Ω resistor using Millman's theorem.

Sol^n

$I = 5 + 3 = 8A$

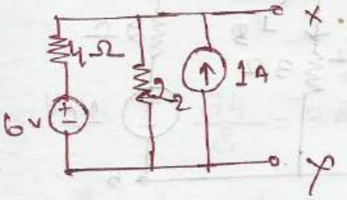
$G = \frac{1}{2} + \frac{1}{2} = 1 ℧$

$V = \frac{I}{G} = 8V$

$R = 1 \Omega$

$I = \frac{V}{R+1} = \frac{8}{2} = 4A$

Q-1

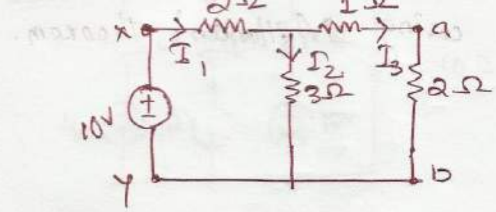


Find Millman's equivalent for the left of the terminals x-y.

6. Reciprocity Theorem.

In any branch of a network, the current (I) due to a single source of voltage (V) elsewhere in the network is equal to the current through the branch in which the source was originally placed when the source is placed in the branch in which the current (I) was originally obtained.

Ex-



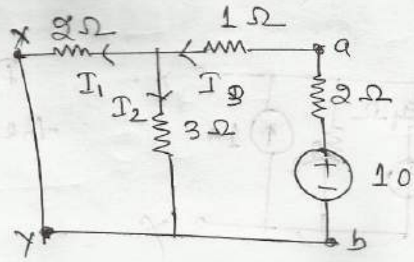
Sol^n

$$R_{eq} = [(2+1) \| 3] + 2$$

$$= 1.5 + 2 = 3.5\Omega$$

$$I_1 = \frac{10}{3.5} = 2.86\,A$$

$$I_2 = I_3 = \frac{2.86}{2} = 1.43\,A$$



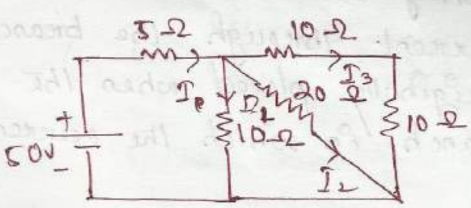
$$R_{eq} = (2\|3) + 1 + 2$$

$$= \frac{6}{5} + 3 = \frac{21}{5} = 4.2\Omega$$

$$I_3 = \frac{10}{4.2} = 2.381\,A$$

$$I_1 = \frac{3}{5} \times 2.381 = 1.43\,A$$

Q



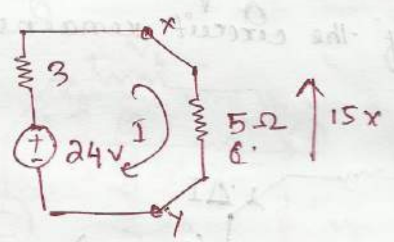
Show the validity of Reciprocity theorem in 5Ω & 20Ω resistance branch

Ans - $I_2 = 1.25\,A$

Substitution Theorem

The voltage across the current through any branch of a dc bilateral network being known, this branch can be replaced by any combination of elements that will make the same voltage across & current through the chosen branch.

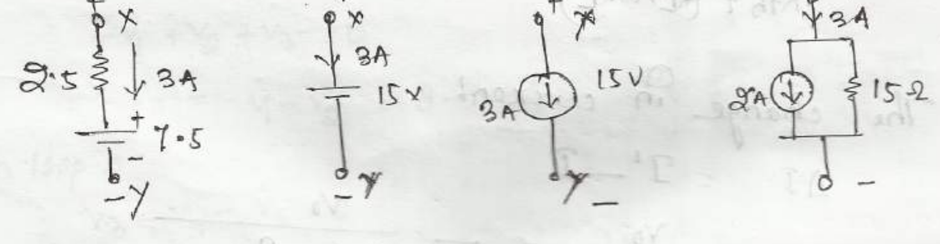
Ex



sol^n

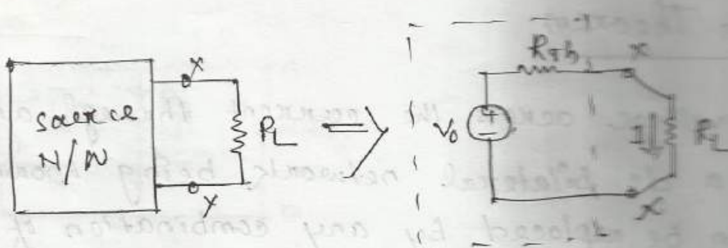
$$I = \frac{24}{8} = 3A$$

The branch x-y can be represented as



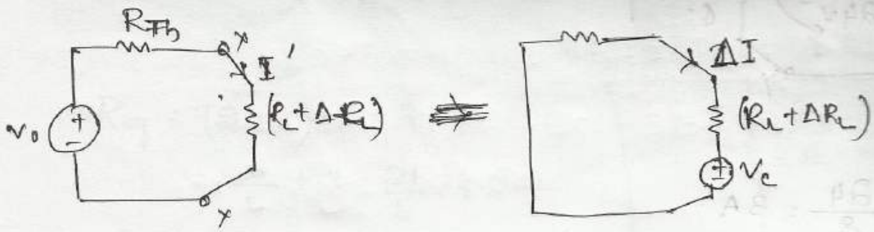
Compensation Theorem

In a linear time-invariant network when the resistance (R) of an uncoupled branch, carrying a current (I), is changed by (ΔR), the currents in all branches would change and can be obtained by assuming that an ideal voltage source of (V_c) has been connected [such that [$V_c = I(\Delta R)$]] in series with (R+ΔR) when all other sources in the N/w are replaced by their internal resistance.



Here $I = \dfrac{V_0}{R_{th}+R_L}$

Let the load resistance R_L be changed to $(R_L+\Delta R_L)$. Since the rest of the circuit remains unchanged,



$$I' = \frac{V_0}{R_{th} + (R_L+\Delta R_L)}$$

The change in current

$$\Delta I = I' - I$$

$$= \frac{V_0}{R_{th}+(R_L+\Delta R_L)} - \frac{V_0}{R_{th}+R_L}$$

$$= \frac{V_0\left[R_{th}+R_L - R_{th} - R_L - \Delta R_L\right]}{R_{th}+(R_L+\Delta R_L)(R_{th}+R_L)}$$

$$= \frac{-V_0\,\Delta R_L}{(R_{th}+R_L)\left[R_{th}+(R_L+\Delta R_L)\right]} = \frac{-I\,\Delta R_L}{R_{th}+R_L+\Delta R_L}$$

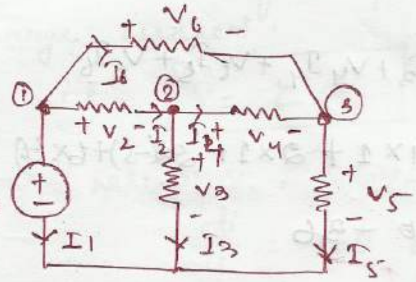
$$\Rightarrow \boxed{\Delta I = \frac{-V_c}{R_{th}+R_L+\Delta R_L}}$$

Pellegen's Theorem

For any given time, the sum of power delivered to each branch of any electric network is zero.

Thus for k^{th} branch, this theorem states that $\sum_{k=1}^{n} v_k i_k = 0$, where 'n' being the no. of branches, v_k the drop in the branch & i_k the through current

Ex



check the validity of Pellegen's theorem, provided $v_1 = 8V$, $v_2 = 4V$, $v_4 = 2V$, $I_1 = 4A$, $I_2 = 2A$, $I_3 = 1A$

Sol

In loop-1

$$-v_1 + v_2 + v_3 = 0$$

$$\Rightarrow v_3 = v_1 - v_2 = 8 - 4 = 4V$$

In loop-2

$$v_3 - v_4 - v_5 = 0$$

$$\Rightarrow v_5 = v_3 - v_4 = 4 - 2 = 2V$$

In loop-3

$$v_2 - v_6 + v_4 = 0$$

$$\Rightarrow v_6 = v_2 + v_4 = 4 + 2 = 6V$$

At node-1

$$I_1 + I_2 + I_6 = 0$$

$$\Rightarrow I_6 = -I_1 - I_2 = -4 - 2 = -6A.$$

At node-2

$I_2 = I_3 + I_4$

$\Rightarrow I_4 = 2 - 1 = 1A$

At node-3

$I_4 + I_6 = I_5$

$\Rightarrow I_5 = 1 - 6 = -5A$

So, the summation of power in the braches.

$\sum V_K I_K = V_1 I_1 + V_2 I_2 + V_3 I_3 + V_4 I_4 + V_5 I_5 + V_6 I_6$

$= 8\times4 + 4\times2 + 4\times1 + 2\times1 + 2\times(-5) + 6\times(-6)$

$= 32 + 8 + 4 + 2 - 10 - 36$

$= 0$

Thus, Tellegen's theorem verified.

Analysis of Coupled Circuits

Coupled circuit – The interconnected loops of an electric network through magnetic fields.

Self Inductance –

When a current changes in a circuit, the magnetic flux linking the same circuit changes (and vice versa) and an emf is induced in the circuit. This induced emf is proportional to the rate of change current.

i.e $$\boxed{v = L\frac{di}{dt}} \quad \text{—(1)}$$

where v = induced voltage

$\frac{di}{dt}$ = Rate of change of current

L = const. of proportionality called self-inductance

Also, we know that

$$\boxed{L = \frac{N\phi}{i}} \quad \text{—— (2)}$$

where N = no. of turns in the circuit

ϕ = flux linkage.

Substituting eqn (2) in eqn(1), we get

$$v = L\frac{d\left(\frac{N\phi}{L}\right)}{dt} = L \times \frac{1}{L} \times N\frac{d\phi}{dt}$$

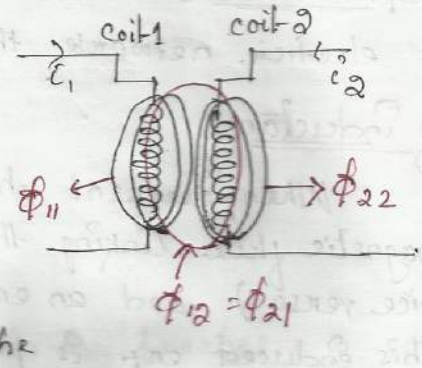
$$\Rightarrow \boxed{v = N\frac{d\phi}{dt}} \quad \text{—— (3)}$$

Comparing eqn (1) & (3)

$$L\frac{di}{dt} = N\frac{d\phi}{dt}$$

$$\Rightarrow \boxed{L = N\frac{d\phi}{di}} \quad \text{—(4)}$$

Mutual Inductance



When two coils carry currents i_1 & i_2 each coil will have leakage flux Φ_{11} & Φ_{22} for coil 1 & coil-2 respectively as well as mutual flux Φ_{21}, the flux of coil-2 links coil-1 & or Φ_{12}, the flux of coil-1 links coil-2.

The induced voltage of coil 2 is

~~$V_{22} = N_2 d\Phi_{21}$~~

$$V_{L2} = N_2 \frac{d\Phi_{12}}{dt} \quad \text{——(1)}$$

Again, since Φ_{12} is related to the current of coil-1 & the induced voltage is proportional to the rate of change of i_1

$$V_{L2} = M \frac{di_1}{dt} \quad \text{——(2)}$$

where M is the mutual inductance between the two coils.

Comparing eqn (1) & (2)

$$M \frac{di_1}{dt} = N_2 \frac{d\Phi_{12}}{dt}$$

$$\Rightarrow \boxed{M = N_2 \frac{d\Phi_{12}}{di_1}} \quad \text{——(3)}$$

Similarly $\boxed{M = N_1 \frac{d\phi_{21}}{di_2}}$ ——(4)

When the coils are linked with air medium, the flux & current are linearly related

So, $M = N_2 \frac{\phi_{12}}{i_1}$
& $M = N_1 \frac{\phi_{21}}{i_2}$ } ——(5)

Coefficient of coupling (K) —

It is defined as the fraction of total flux that links the coil.

i.e $\boxed{K = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}}$

The maximum value of K is unity.

From eqn (3)

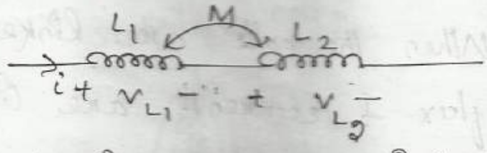
$$M^2 = N_1 N_2 \frac{\phi_{21}\phi_{12}}{i_1 i_2} = N_1 N_2 \frac{K\phi_1 K\phi_2}{i_1 i_2}$$

$$= K^2 \frac{N_1 \phi_1}{i_1} \frac{N_2 \phi_2}{i_2}$$

$$\Rightarrow M^2 = K^2 L_1 L_2$$

$$\Rightarrow \boxed{M = K\sqrt{L_1 L_2}}$$

Series Connection of coupled coils



When two coils of self-inductance L_1 & L_2 are connected in series having mutual inductance M.

then $$v_{L_1} = L_1\frac{di}{dt} + M\frac{di}{dt} = (L_1+M)\frac{di}{dt} \quad —(1)$$

$$v_{L_2} = L_2\frac{di}{dt} + M\frac{di}{dt} = (L_2+M)\frac{di}{dt} \quad —(2)$$

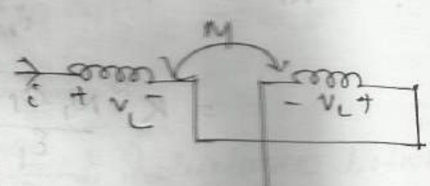
Net voltage $v_L = v_{L_1} + v_2$

$$\Rightarrow v_L = (L_1 + M + L_2 + M)\frac{di}{dt}$$

$$\Rightarrow v_L = (L_1 + L_2 + 2M)\frac{di}{dt} \quad —(3)$$

So, the total inductance

$$\boxed{L = L_1 + L_2 + 2M}$$



When the coils are connected in series but the flux of both the coils oppose each other.

i.e $$v_{L_1} = (L_1 - M)\frac{di}{dt}$$

$$v_{L_2} = (L_2 - M)\frac{di}{dt}$$

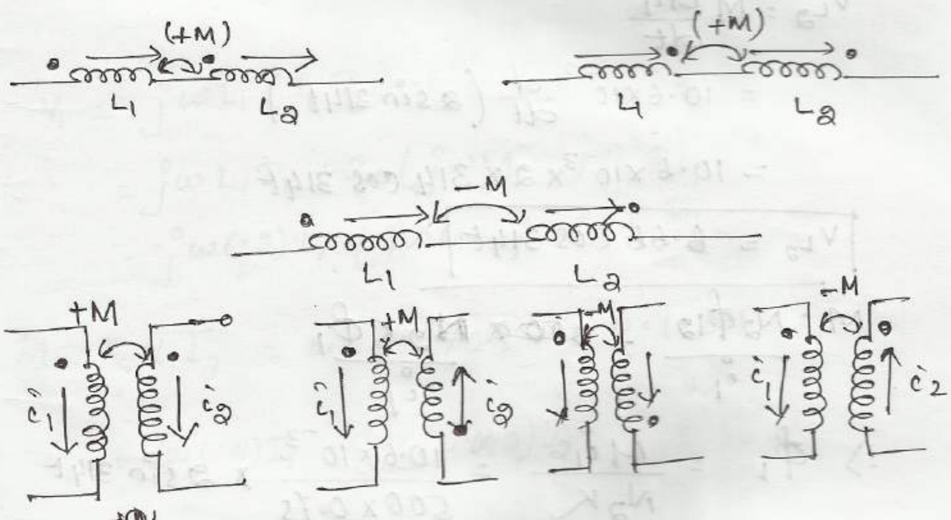
The net inductance is

$$\boxed{L = L_1 + L_2 - 2M}$$

Dot convention in coupled circuit

To determine the relative polarity of the induced voltage in the coupled coil, the coils are marked with dots. On each coil, a dot is placed at the terminals.

When the currents through each of the mutually coupled coils are going away from the dot or towards the dot, the mutual inductance is +ve while for the case when the current through the coil is leaving the dot for one coil & entering the other, the mutual inductance is −ve.



Ex-1 Two coupled coils have self-inductances $L_1 = 10\times10^{-3}H$ & $L_2 = 20\times10^{-3}H$. The coefficient of coupling (K) being 0.75 in the air, find voltage in the 2nd coil & the flux of first coil provided the 2nd coil has 500 turns & the circuit current is given by $i_1 = 2\sin 314t$ A.

Soln

$$M = K\sqrt{L_1 L_2}$$

$$= 0.75\sqrt{10\times10^{-3}\times20\times10^{-3}}$$

$$= 10.6\times10^{-3}H$$

$$V_{L_2} = M\frac{di_1}{dt}$$

$$= 10.6\times10^{-3}\frac{d}{dt}(2\sin 314t)$$

$$= 10.6\times10^{-3}\times2\times314\cos 314t$$

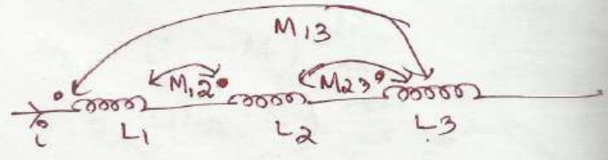
$$\boxed{V_{L_2} = 6.66\cos 314t}$$

$$M = \frac{N_2\phi_{12}}{i_1} = \frac{N_2 K\phi_1}{i_1}$$

$$\Rightarrow \phi_1 = \frac{Mi_1}{N_2 K} = \frac{10.6\times10^{-3}}{500\times0.75}\times2\sin 314t$$

$$\Rightarrow \boxed{\phi_1 = 5.66\times10^{-5}\sin 314t}$$

Ex-2



Find the total inductance of the coupled ckt.
Given $L_1 = 1H$ $L_2 = 2H$ $L_3 = 5H$
$M_{12} = 0.5H$ $M_{23} = 1H$ $M_{13} = 1H$

sol^n

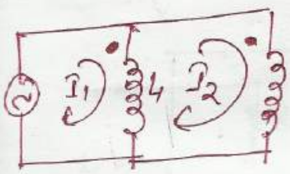
For coil 1 – $L_1 + M_{12} + M_{13} = 1 + 0.5 + 1 = 2.5H$

For coil 2 – $L_2 + M_{12} + M_{23} = 2 + 0.5 + 1 = 3.5H$

For coil 3 – $L_3 + M_{23} + M_{13} = 5 + 1 + 1 = 7H$.

The net inductance $= 2.5 + 3.5 + 7 = 13H$

Ex-5



Find the input impedance as well as the net inductance.

$L_1 = 0.2H \quad L_2 = 0.5H$ & $K = 0.5$

sol^n

$$V_1 = j\omega L_1 (I_1 - I_2) + j\omega M I_2$$

$$= j\omega L_1 I_1 + I_2 (j\omega M - j\omega L_2)$$

$$= j\omega (.2) I_1 + [j\omega M - j\omega (.5)] I_2$$

$$M = K\sqrt{L_1 L_2} = 0.5\sqrt{.2 \times 0.5} = 0.158H$$

So, $V_1 = j\omega(.2) I_1 - j\omega (0.042) I_2$

Again $j\omega L_1 (I_2 - I_1) + j\omega L_2 I_2 + j\omega M I_1 = 0$

$\Rightarrow j\omega I_1 (M - L_1) + j\omega I_2 (L_1 + L_2) = 0$

$\Rightarrow j\omega I_1 (-0.042) + j\omega I_2 (0.7) = 0$

$\Rightarrow I_2 = \dfrac{0.042}{0.7} I_1$

So, $V_1 = j\omega(0.2)I_1 - 0.042 \times j\omega \times \frac{0.042}{0.7} I_1$

$= j\omega I_1 (0.2 - 0.00252)$

$= j\omega (0.1975) I_1$.

So, $L_{equivalent} = 0.1975\ H$

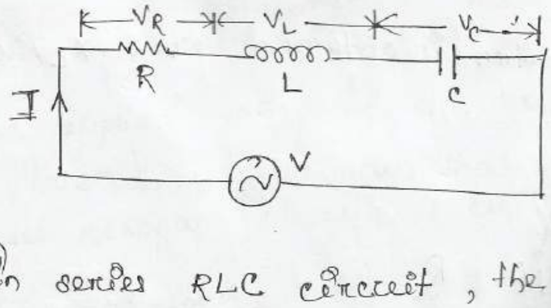
$Z_{in} = \frac{V_1}{I_1} = j\omega(0.1975)\ \Omega$

Resonance and Selectivity

Resonance

Resonance in electrical circuits represent consisting of passive and active elements represents a particular state of the circuit when current or voltage in the circuit is maximum or minimum with respect to the magnitude of excitation at a particular frequency, the circuit impedance being either minimum or maximum at the power factor unity.

Series Resonance



In series RLC circuit, the circuit current I is given by

$$\boxed{I = \frac{V}{Z}}$$

where Z represents the equivalent impedance of the circuit.

$$Z = R + j\omega L + \frac{1}{j\omega C}$$

$$= R + j\omega L - \frac{j}{\omega C}$$

$$\Rightarrow \boxed{Z = R + j(X_L - X_C)} \quad \text{——— (1)}$$

The expression of frequency of resonance can be obtained as :-

Let f_0 or ω_0 be the frequency at which $X_L = X_C$

i.e $\omega_0 L = \frac{1}{\omega_0 C}$

$\Rightarrow \omega_0^2 = \frac{1}{LC}$

$$\Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec}}$$

$$\Rightarrow \boxed{f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz.}}$$

Properties of Resonance of RLC Series Circuit.

1) The applied voltage & the resulting current are in phase which also mean that the P.f of the RLC series resonant circuit is unity.

2) The net reactance is zero at resonance & the impedance does have the resistive part only.

3) The current in the circuit is maximum & is (V/R) A.

4) At resonance, the circuit has got minimum impedance & maximum admittance.

5) Frequency of resonance is given by $f_0 = \frac{1}{2\pi\sqrt{LC}}$ Hz.

Q- Factor of Series Resonating Circuit

In a series resonating circuit, Q factor (Quality factor) is defined as the ratio of the voltage across the inductor or capacitor to the applied voltage.

i.e. $$\boxed{Q = \frac{V_L}{V} = \frac{V_C}{V}}$$

where V_L is the voltage across the inductor, V_C the voltage across the capacitor at resonance & V the applied voltage.

$$Q = \frac{V_L}{V} = \frac{I_0 X_L}{I_0 R} = \frac{X_L}{R} = \frac{\omega_0 L}{R} \text{ [for coil]}$$

Also $$Q = \frac{V_C}{V} = \frac{I_0 X_C}{I_0 R} = \frac{X_C}{R} = \frac{1}{\omega_0 C R} \text{ [for capacitor]}$$

Again $$V_L = I_0 X_L = \frac{V}{R} X_L = \frac{V}{R} \omega_0 L = \frac{\omega_0 L}{R} V$$

$$\Rightarrow \boxed{V_L = Q \text{ factor} \times V} \text{ volts}$$

and $$V_C = I_0 X_C = \frac{V}{R} \times \frac{1}{\omega_0 C} = \frac{1}{\omega_0 R C} \cdot V$$

$$\Rightarrow \boxed{V_C = Q \text{ factor} \times V} \text{ volts}$$

Also $$Q = \frac{\omega_0 L}{R} = \frac{1}{\sqrt{LC}} \cdot \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Also $$Q = \frac{1}{\omega_0 R C} = \frac{1}{\frac{1}{\sqrt{LC}} R C} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Bandwidth of Series Resonating Circuit & its Relation with Q

The frequency band within the limits of lower and upper half power frequency is called the bandwidth of the resonant circuit.

A half power frequencies, the net reactance of the series resonant circuit is R and given by

$$\boxed{X = \pm(X_L - X_C) = R}$$

i.e $R = \pm\left(\omega L - \frac{1}{\omega C}\right) = \pm X$

Let f_1 be the frequency when the net circuit reactance be -ve and f_2 be frequency when the net circuit reactance is +ve.

Thus $\left(\omega_2 L - \frac{1}{\omega_2 C}\right) = R$ ——— (1)

and $\left(\omega_1 L - \frac{1}{\omega_1 C}\right) = -R$ ——— (2)

Adding eq's (1) & (2)

$$(\omega_2 + \omega_1)L - \frac{1}{C}\left(\frac{1}{\omega_2} + \frac{1}{\omega_1}\right) = 0$$

$$\Rightarrow (\omega_2 + \omega_1)L - \frac{1}{C}\left(\frac{\omega_1 + \omega_2}{\omega_1 \omega_2}\right) = 0$$

$$\Rightarrow L = \frac{1}{C} \cdot \frac{1}{\omega_1 \omega_2}$$

$\Rightarrow$ [illegible]

Again subtracting eq^n (2) from eq^n (1)

$$(\omega_2-\omega_1)L + \frac{1}{C}\left(\frac{1}{\omega_1}-\frac{1}{\omega_2}\right) = 2R$$

$$\Rightarrow (\omega_2-\omega_1)L + \frac{1}{C}\left(\frac{\omega_2-\omega_1}{\omega_2\omega_1}\right) = 2R \quad \text{——— (3)}$$

dividing eq^n (3) by L

$$(\omega_2-\omega_1) + \frac{1}{LC}\left(\frac{\omega_2-\omega_1}{\omega_2\omega_1}\right) = \frac{2R}{L}$$

$$\Rightarrow (\omega_2-\omega_1) + \frac{1}{LC}\,\frac{\omega_2-\omega_1}{\frac{1}{LC}} = \frac{2R}{L}$$

$$\Rightarrow 2(\omega_2-\omega_1) = \frac{2R}{L}$$

$$\Rightarrow \boxed{(\omega_2-\omega_1) = \frac{R}{L}} \quad \text{——— (4)}$$

$$\Rightarrow \boxed{(f_2-f_1) = \frac{1}{2\pi}\left(\frac{R}{L}\right)}$$

We know that

$$Q = \omega_0 \frac{L}{R} \Rightarrow \frac{R}{L} = \frac{\omega_0}{Q} \quad \text{——— (5)}$$

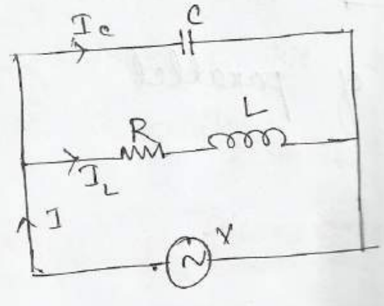
Comparing with eq^n (4)

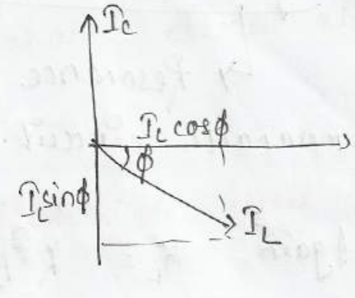
$$\frac{\omega_0}{Q} = \omega_2-\omega_1$$

$$\Rightarrow \boxed{Q = \frac{\omega_0}{\omega_2-\omega_1} = \frac{f_0}{f_2-f_1}}$$

$$\text{ie} \quad \boxed{Q = \frac{f_0}{\text{Bandwidth}} = \frac{\text{Resonant frequency}}{\text{Bandwidth}}}$$

Parallel Resonance





$$Y = Y_1 + Y_2$$

$$= \frac{j}{X_C} + \frac{1}{R + jX_L}$$

$$= \frac{j}{X_C} + \frac{R - jX_L}{R^2 + X_L^2}$$

$$= \frac{R}{R^2 + X_L^2} + j\left(\frac{1}{X_C} - \frac{X_L}{R^2 + X_L^2}\right)$$

At resosance the imaginary part must be zero.

i.e $\frac{1}{X_C} = \frac{X_L}{R^2 + X_L^2}$

$$\Rightarrow \omega_0 C = \frac{\omega_0 L}{R^2 + \omega_0^2 L^2}$$

$$\Rightarrow R^2 + \omega_0^2 L^2 = \frac{L}{C}$$

$$\Rightarrow \omega_0^2 L^2 = \frac{L}{C} - R^2$$

$$\Rightarrow \omega_0^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\Rightarrow \omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$\Rightarrow \boxed{\omega_0 = \frac{1}{L}\sqrt{\frac{L}{C} - R^2}}$$

$$f_0 = \frac{1}{2\pi L}\sqrt{\frac{L}{C} - R^2}$$

↳ Resonance frequency of parallel resonance circuit.

Again $Z_L = R + j\omega L$

$$Z_L = \sqrt{R^2 + (\omega_0 L)^2}$$

$$= \sqrt{R^2 + \frac{L}{C} - R^2}$$

$$\Rightarrow Z_L = \sqrt{\frac{L}{C}}.$$

At resonance

$$Y = \frac{R}{R^2 + X_L^2}$$

$$Z = \frac{R^2 + X_L^2}{R} = \frac{R^2 + \omega_0^2 L^2}{R}$$

$$= \frac{R^2 + \frac{L}{C} - R^2}{R}$$

$$\Rightarrow Z = \frac{L}{CR}$$

So, the resonance current is

$$I = \frac{V}{L/CR}.$$

Note

As $I_L \sin\phi$ and I_C cancels each other, hence the power factor of this parallel resonance circuit at resonce is unity.

Properties of Resonance of parallel LRC circuit

1) Power factor is unity.

2) Current as resonance is $\left[\frac{V}{L/CR}\right]$ and is in phase with the applied voltage. The value of current at resonce is minimum.

3) Net impedance at resonance of the parallel circuit is maximum i.e $(L/CR)\ \Omega$.

4) The admittance is minimum at responance & the net susceptance is zero at resonance.

6) The resonance frequency of this circuit is

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

Q-Factor

Q. factor of a parallel circuit is the current magnification of the circuit at resonance. It represents the ratio of the current circulating between the two parallel branch,

i.e $Q = \frac{I_C}{I} = \frac{V/X_C}{V/Z} = \frac{Z}{X_C}$

$$\Rightarrow Q = \frac{L}{CR} \times \frac{1}{\frac{1}{\omega_0 C}}$$

$$= \frac{L}{CR} \times \omega_0 C$$

$$\Rightarrow \boxed{Q = \frac{\omega_0 L}{R}}$$

In parallel circuit

$$\omega_0 = \frac{1}{L}\sqrt{\frac{L}{C} - R^2}$$

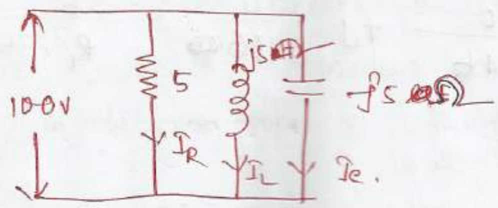
If the resistor is neglected

then $\omega_0 = \frac{1}{\sqrt{LC}}$

So, $Q = \frac{1}{\sqrt{LC}} \frac{L}{R}$

$$\Rightarrow \boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

Q. Determine all the current of the given ckt at resonance condⁿ.



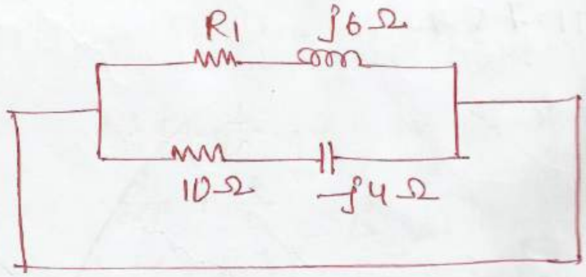
Solⁿ

$$I_R = \frac{V}{R} = \frac{100}{5} = 20\angle 0^\circ \text{ amp.}$$

$$I_L = \frac{V}{X_L} = \frac{100\angle 0^\circ}{5\angle 90^\circ} = 20\angle -90^\circ$$

$$I_C = \frac{100\angle 0^\circ}{5\angle -90^\circ} = 20\angle 90^\circ$$

Q.



Find R_1.

Solⁿ

$$Y = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$= \frac{(R_1 - j6)}{(R_1 + j6)(R_1 - j6)} + \frac{100 + j4}{(100 - j4)(100 + j4)}$$

$$= \frac{R_1 - j6}{R_1^2 + 36} + \frac{100 + j4}{100 + 16}$$

$$\Rightarrow \frac{R_1}{R_1^2+36} - \frac{j6}{R_1^2+36} + \frac{10}{100+16} + \frac{j4}{100+16}$$

$$\Rightarrow Y_1 = \frac{R_1}{R_1^2+36} + \frac{10}{116} + j\left(\frac{4}{116} - \frac{6}{R_1^2+36}\right)$$

At resonance

$$\frac{4}{116} - \frac{6}{R_1^2+36} = 0$$

$$\Rightarrow \frac{3}{R_1^2+36} = \frac{2}{116}$$

$$\Rightarrow R_1^2+36 = \frac{3 \times 58}{1} = 174$$

$$\Rightarrow R_1^2 = 138$$

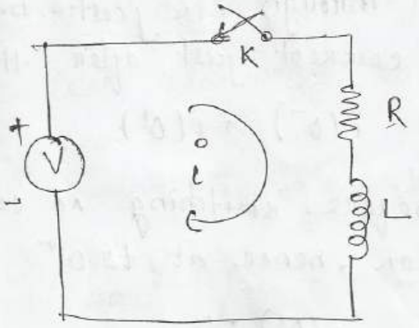
$$\Rightarrow R_1 = 11.75\ \Omega$$

Transient Response of Passive Circuits

Any disturbance or sudden change in applied voltage from one finite value to another is known as transient. and that transient occurs between two steady state conditions.

Transient Response of series R-L circuit having DC excitation

Let a dc voltage V be applied suddenly (i.e at $t=0$) by closing a switch K in a series R-L circuit.



Applying Kirchhoff's voltage law

$$Ri + L\frac{di}{dt} = V$$

$$\Rightarrow \frac{di}{dt} + \frac{R}{L}i = \frac{V}{L}$$

$$\Rightarrow \left(P+\frac{R}{L}\right)i = \frac{V}{L} \quad \left[\text{where } P = \frac{d}{dt}\right] \quad \text{--- (1)}$$

It is a non-homogeneous differential eq^n. So, it contain a complementary function & a particular sol^n,

i.e $i = i_c + i_p$.

$$i_c = ce^{-(R/L)t}$$

$$i_p = e^{-(R/L)t}\int e^{(R/L)t}\left(\frac{V}{L}\right)dt$$

$$= \frac{V}{L} \times \frac{L}{R} = \frac{V}{R}$$

So, $\boxed{i = ce^{-(R/L)t} + \frac{V}{R}}$ —— (2)

An inductance, due to its "electrical inertia" does not allow sudden change of current through it following the rules of electromagnetic induction & hence at

Note

$y' + ay = b$

$I.F = e^{\int a\,dx} = e^{ax}$

$y \cdot I.F = \int b \times I.F$

$\Rightarrow y e^{ax} = \int b e^{ax} dx$

$y e^{ax} = \frac{b}{a} e^{ax} + C$

$\boxed{y = \frac{b}{a} + Ce^{-ax}}$

current through it just before switching is same to the current just after the switching

ie $\quad i(0^-) = i(0^+)$

As, before switching no current through the inductor, hence at $t = 0^+$

$$i(0^+) = 0$$

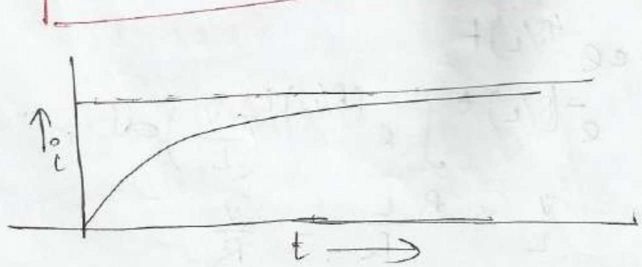
So, at initial condition at $t = 0^+$

$$0 = ce^{-(R/L)0} + \frac{V}{R}$$

$$\Rightarrow \boxed{c = -\frac{V}{R}}$$

So, $\quad i = -\frac{V}{R} e^{-(R/L)t} + \frac{V}{R}$

$$\boxed{i = \frac{V}{R}\left(1 - e^{-(R/L)t}\right) \text{ A}} \quad \text{—— (3)}$$



The steady state current obtain at $t = \frac{L}{R}$.

i.e $i = \frac{V}{R}\left(1-e^{-1}\right) = \frac{V}{R}(1-.368)$

$\Rightarrow \boxed{i = 0.632 \frac{V}{R}}$

So, at time $t = \frac{L}{R}$, the current through the R-L circuit rises to 63.2% of the final value. This time is known as "time constant" (TC) and the inverse $\left(\frac{R}{L}\right)$ is called damping ratio.

$$V_R = iR = V\left(1-e^{-(R/L)t}\right) \quad \text{--- (4)}$$

$$V_L = L\frac{di}{dt} = L\frac{d}{dt}\left[\frac{V}{R}\left(1-e^{-R/L\,t}\right)\right]$$

$$\Rightarrow V_L = \frac{LV}{R}\,\frac{e^{-(R/L)t}}{\left(\frac{R}{L}\right)} \times \frac{R}{L}$$

$$\Rightarrow \boxed{V_L = V e^{-(R/L)t}} \quad \text{--- (5)}$$

Let us now analyse another transient condition of the R-L circuit assuming that following the closing of the switch, the circuit reaches at steady state (at $t = \infty$) and suddenly the voltage is withdrawn by opening the switch K & throwing it to K'.

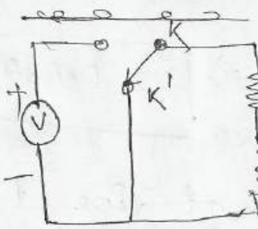
$$Ri + L\frac{di}{dt} = 0$$

$$\Rightarrow \left(P + \frac{R}{L}\right)i = 0$$

Here $i_p = 0$

$$i = i_c + 0$$

$$\Rightarrow i = i_c = c' e^{-(R/L)t} \quad \text{———} (6)$$

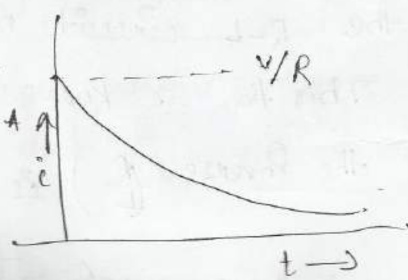


$$i(0^+) = \frac{V}{R} \quad \text{at } t = 0$$

So, $$\frac{V}{R} = c' e^{-(R/L)\cdot 0}$$

$$\Rightarrow c' = \frac{V}{R}$$

Hence, $$\boxed{i = \frac{V}{R} e^{-(R/L)t}}$$



$$V_R = iR = V e^{-(R/L)t}$$

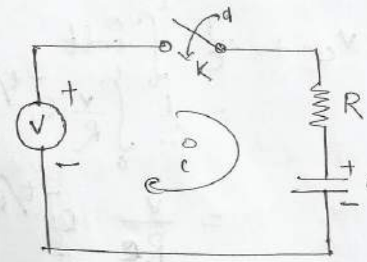
$$V_L = L\frac{di}{dt} = L\frac{d}{dt}\left(\frac{V}{R} e^{-R/L\,t}\right)$$

$$= L\frac{V}{R} e^{-R/L\,t}\cdot\left(-\frac{R}{L}\right)$$

$$\Rightarrow \boxed{V_L = -V e^{-(R/L)t}}$$

Transient Response in series R.C Circuit with

When switch is on



$$V = Ri + \frac{1}{C}\int i\,dt \quad \text{——(1)}$$

Differentiating eqn (1)

$$R\frac{di}{dt} + \frac{i}{C} = 0 \quad \text{——(2)}$$

$$\Rightarrow \frac{di}{dt} + \frac{1}{RC}i = 0$$

The above eqn contain only complementary function.

So, the soln is

$$i = i_c = Ke^{-t/RC} \quad \text{——(3)}$$

With application of voltage and no initial charge across the capacitor, the capacitor will not produce any voltage across it but acts as a short circuit causing the current to be (V/R).

i.e at $t = 0^+$, $i(0^+) = V/R$

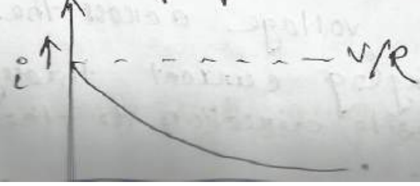
So, eqn (3) becomes

$$\frac{V}{R} = K \quad \text{——(4)}$$

Hence

$$\boxed{i = \frac{V}{R}e^{-t/RC}} \quad \text{——(5)}$$

So, the charging current is a decaying function



$$v_R = iR = V e^{-t/RC}$$

& $v_c = \frac{1}{C}\int_0^t i\,dt$

$$= \frac{1}{C}\int_0^t \frac{V}{R} e^{-t/RC}\,dt$$

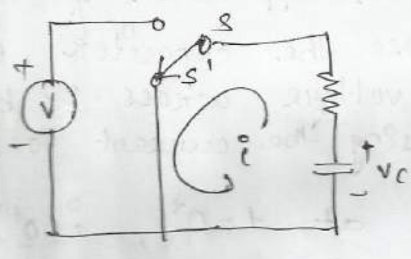
$$= \frac{V}{RC} e^{-t/RC} \times (RC)$$

$$\Rightarrow v_c = V\left[-e^{-t/RC}\right]_0^t$$

$$\Rightarrow \boxed{v_c = V\left(1 - e^{-t/RC}\right)} \quad \text{—— (6)}$$

Discharging condition

In the discharging case



$$Ri + \frac{1}{C}\int i\,dt = 0 \quad \text{—— (6)}$$

Differentiating above eq^n

$$R\frac{di}{dt} + \frac{i}{C} = 0$$

$$\Rightarrow \left(P + \frac{1}{RC}\right) i = 0 \quad \text{—— (7)}$$

Sol^n of the above eq^n is

$$i = K' e^{-t/RC} \quad \text{—— (8)}$$

Now $t = 0^+$, the voltage across the capacitor will start discharging current through resistor in opposite direction to the original current direction.

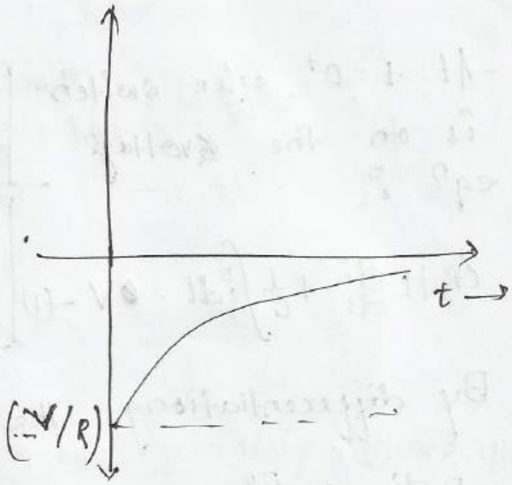
i.e $i(0^+) = -\frac{V}{R}$

So, at $t = 0^+$

$$-\frac{V}{R} = K' e^{0}$$

So, eq^n (8) becomes

$$\boxed{i = -\frac{V}{R} e^{-t/RC}}$$

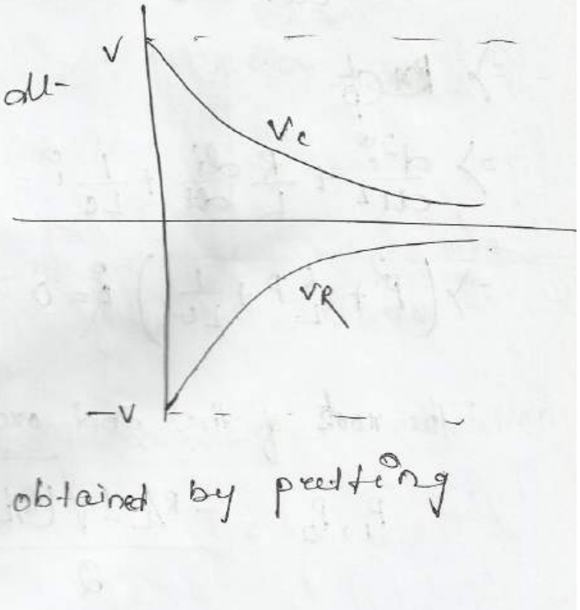


$$\boxed{V_R = iR = -V e^{-t/RC}}$$

$$V_C = \frac{1}{C}\int i\,dt$$

$$= -\frac{V}{RC}\int_0^t e^{-t/RC} \times (-RC)\, dt$$

$$\boxed{V_C = V \; e^{-t/RC}} \quad (10)$$



The time constant is obtained by putting $t = RC$.

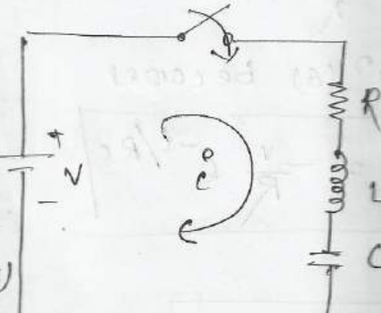
So, eq^n(6) become

$$i = V(1 - e^{-1}) = .632V$$

So, at time constant $(\tau = RC)$ the capacitor attains 63.2% of steady state voltage.

Transient response in RLC circuit with DC Excitation

At $t = 0^+$, after switch is on the voltage eq^n is



$$iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = V \quad -(1)$$

By differentiating the above eq^n

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}i = 0$$

$$\Rightarrow \frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0$$

$$\Rightarrow \left(P^2 + \frac{R}{L}P + \frac{1}{LC}\right)i = 0 \quad \text{------- (2)}$$

The roots of the 2nd order eq^n is

$$P_1, P_2 = \frac{-R/L \pm \sqrt{(R/L)^2 - 4/LC}}{2}$$

Let $\alpha = (-R/2L)$

$$\beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Hence $P_1 = \alpha + \beta$

$P_2 = \alpha - \beta$

So, the solⁿ of eqⁿ (2) becomes

$$i = C_1 e^{P_1 t} + C_2 e^{P_2 t} \quad \text{---} (3)$$

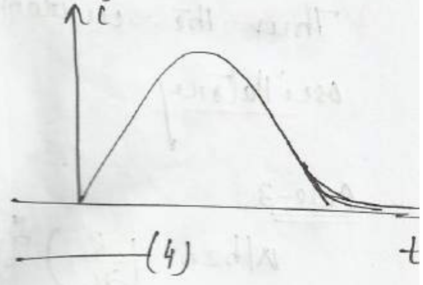
case-1

When $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$

In this condition β is +ve real quantity. Hence, the roots P_1 & P_2 are real but unequal.

i.e $P_1 = \alpha + \beta$ & $P_2 = \alpha - \beta$

So, $i = C_1 e^{(\alpha+\beta)t} + C_2 e^{(\alpha-\beta)t}$



$$\Rightarrow i = e^{\alpha t}\left(C_1 e^{\beta t} + C_2 e^{-\beta t}\right) \quad \text{---} (4)$$

This condition is called over-damped condition.

case-2

When $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$

In this condition β is imaginary, hence, the roots P_1 & P_2 are complex conjugates.

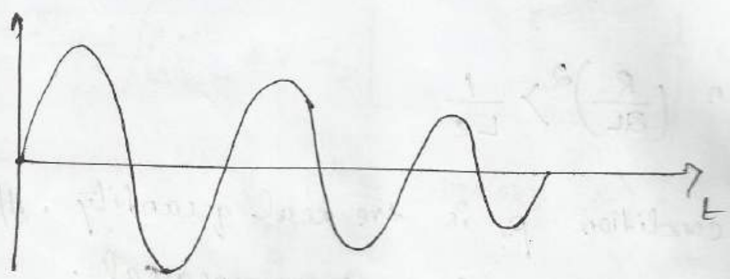
ie $P_1 = \alpha + j\beta$

$P_2 = \alpha - j\beta$

So, $i = C_1 e^{(\alpha+j\beta)t} + C_2 e^{(\alpha-j\beta)t}$

$$= e^{\alpha t}\left(C_1 e^{j\beta t} + C_2 e^{-j\beta t}\right)$$

$$i = e^{\alpha t}\left[C_1 \cos\beta t + C_1 \sin\beta t\right]$$



Thus the current sol^n is underdamped or oscillatory

Case-3

When $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$

In this condition $\beta = 0$

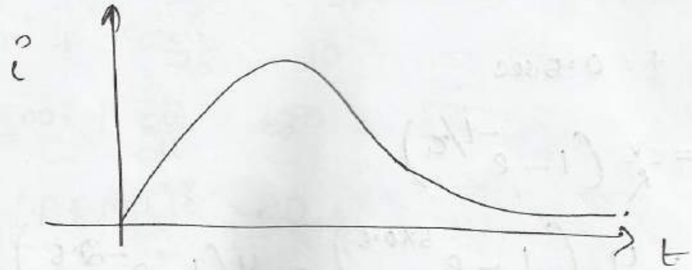
Hence the roots

$$P_1 = P_2 = \alpha$$

and $i = C_1 e^{\alpha t} + C_2 t e^{\alpha t}$

$$i = e^{\alpha t}\left(C_1 + C_2 t\right)$$

So, the current response is critically damped.



Q A series R-L circuit has $R = 25\,\Omega$ & $L = 5H$. A dc voltage of 100 V is applied at $t = 0$. Find (a) the eq^n's for charging current, voltage across R & L (b) the current in the circuit 0.5 sec later & (c) the time at which the drop across R & L are same.

Sol^n

Time constant $(\tau) = \frac{L}{R} = \frac{5}{25} = \frac{1}{5}$ sec.

The charging current

$$i = \frac{V}{R}\left(1 - e^{-t/\tau}\right)$$

$$= \frac{100}{25}\left(1 - e^{-5t}\right)$$

$$\Rightarrow \boxed{i = 4\left(1 - e^{-5t}\right)\ A}$$

Voltage drop across R is

$$\boxed{V_R = iR = 100\left(1 - e^{5t}\right)\ V}$$

$$v_L = L\frac{di}{dt} = V e^{-t/\tau} = 100\, e^{-5t}\ V$$

(b) At $t = 0.5\,sec$

$$i = \frac{V}{R}\left(1 - e^{-t/\tau}\right)$$

$$= 4\left(1 - e^{-5\times 0.5}\right) = 4\left(1 - e^{-2.5}\right)$$

$$\Rightarrow \boxed{i = ~~[illegible]~~\ 3.67\ A}$$

(C) To satisfy the condition of $V_R = V_L = 50\ V$, since applied voltage is 100V.

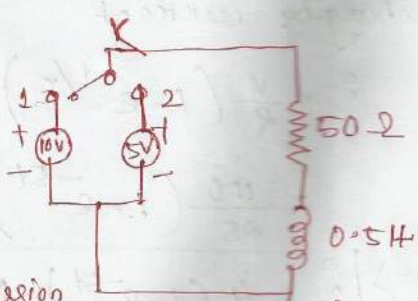
$$50 = L\frac{di}{dt} = V e^{-t/\tau}$$

$$\Rightarrow \frac{1}{2} = e^{-5t}$$

$$\Rightarrow \boxed{t = 0.139\ sec}$$

Q

In the fig., the switch is closed at position 1 at $t = 0$. At $t = 0.5\,m\,sec$, the switch is moved to position 2. Find the expression for the current in both the conditions & sketch the transient.



Solⁿ — At position 1 of the switch

$$50i + 0.5\frac{di}{dt} = 10$$

$$\Rightarrow 100i + \frac{di}{dt} = 20$$

$$\Rightarrow (P+100)i = 20$$

$$\frac{R}{L} = \frac{50}{.5} = 100$$

$$i = c\,e^{-R/L\,t} + \frac{V}{R}$$

$$\Rightarrow \boxed{i = c\,e^{-100t} + .2}$$

At $t = 0$, $i = 0$

$$0 = c + 0.2 \quad \Rightarrow \boxed{c = -0.2}$$

$$\boxed{i = .2\left(1 - e^{-100t}\right)}\ A$$

At $t = 0.5$ msec

$$i = 0.2\left(1 - e^{-100 \times 5 \times 10^{-3}}\right) = 9.754 \text{ mA}$$

At switch position 2, the eqⁿ is

$$50i + 0.5\frac{di}{dt} = 5$$

$$\Rightarrow \frac{di}{dt} + 100i = 10$$

$$\Rightarrow (P+100)i = 10$$

$$i = c' e^{-R/L\,t} + \frac{V}{R}$$

$$\frac{R}{L} = 100$$

$$i = c' e^{-100(t-t')} + .1$$

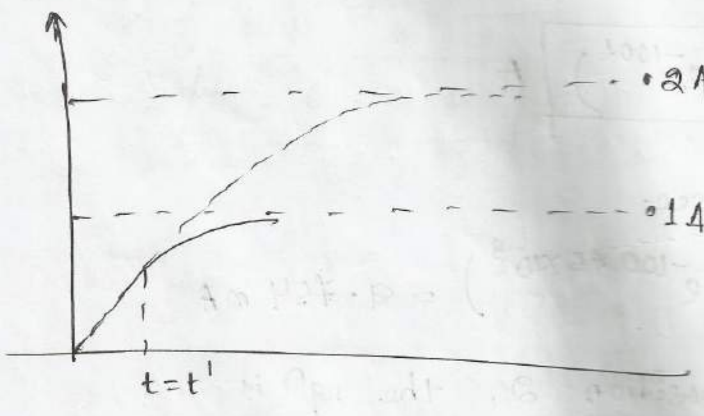
$t' = .5$ msec.

At $t' = .5$ msec $i = 9.75$ mA

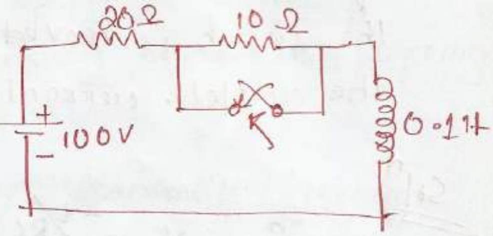
So, $9.75 = c' + .1$

$\Rightarrow c' = -0.09$

So, $$\boxed{i = -0.09\, e^{-100(t-t')} + .1}\ A$$



Q A d.c voltage of 100V is applied in the adjoining circuit and the switch K is open. The switch K is closed at t=0. Find the complete expression for the current.



Sol^n The switch is closed at t=0
the mesh eq^n is

$$100 = 20i + 0.1\frac{di}{dt}$$

$$\Rightarrow (P+200)\,i = 1000$$

$$i = C\,e^{-R/L\,t} + \frac{V}{R}$$

$$i = C\,e^{-200t} + 5$$

Before, switch is on, the steady state current in the circuit is $\frac{V}{R} = \frac{100}{30} = 3.33\,A$.

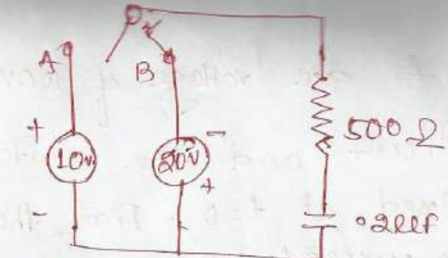
So, at t=0 sec.

$$3.33 = C + 5 \Rightarrow C = -1.67$$

Hence the complete eq^n

$$\boxed{i = -1.67\,e^{-200t} + 5}$$

Q The switch K is closed at position A at t=0. After the lapse of time equivalent to one time constant, the switch is moved to position B. Determine the complete current.



Sol^n

$$i = \frac{V}{R} e^{-t/RC}$$

$$\tau = RC = 500 \times \cdot 2 \times 10^{-6} = 10^{-4} \text{ sec.}$$

$$i = \frac{10}{500} e^{-10000t}$$

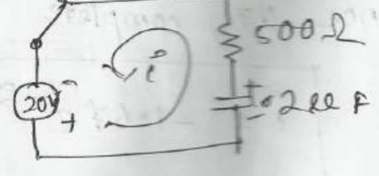
$$\Rightarrow \boxed{i = \cdot 02\, e^{-10000t} \text{ A}}$$

this current will cont. till one time const. when switch is moved to position B.

$$v_c = V\left(1 - e^{-1}\right) = \cdot 632 \times 10 = 6\cdot 32 \text{ V}$$

Case-2

$$i = K' e^{-10000(t-t')}$$



At $t = t'$

$$i = -(20+6\cdot 32)/500 = -\cdot 04 \text{ A}$$

So, $-\cdot 04 = K' \Rightarrow \boxed{K' = -0\cdot 04}$

$$\boxed{i = -0\cdot 04\, e^{-10000(t-t')}}$$

MODULE- II

LAPLACE TRANSFORM

Definition:

Given a function *f (t)*, its Laplace transform, denoted by *F(s)* or *L*[*f (t)*], is given by,

$$L[f(t)] = F(s) = \int_{-\infty}^{\infty} f(t)\, e^{-st} dt$$

where, *s* is a complex variable given by, $s = \sigma + j\omega$

***** The *Laplace transform* is an integral transformation of a function *f* (*t*) from the time domain into the complex frequency domain, giving *F*(*s*).

Properties of L.T.

(i) Multiplication by a constant:-

Let, K be a constant

F(s) be the L.T. of f(t)

Then; $L[kf(t)] = \int_0^{\infty} kf(t)e^{-st}dt = k\int_0^{\infty} f(t)\, e^{-st}dt = kF(S)$

(ii) Sum and Difference:-

Let $F_1(S)$ & $F_2(S)$ are the L.T. of the functions $f_1(t)$& $f_2(t)$ respectively.

$$L[f_1(t) \pm f_2(t)] = F_1(S) + F_2(S)$$

(iii) Differentiation w.r.t. time [Time – differentiation]

$$L\left[\frac{df(t)}{dt}\right] = SF(S) - f(0^+)$$

Proof

F(S)=$L[f(t)] = \int_0^{\infty} f(t)e^{-st}dt$

Let, f(t)=u; then, $\frac{df(t)}{dt}dt = du$

& $e^{-st}dt = dv => v = \frac{-e^{-st}}{s}$

So, $\int_0^{\infty} f(t)e^{-st}dt$= - $\int_0^{\infty} \frac{-e^{-st}}{s} du + f(t)\left(\frac{-e^{-st}}{s}\right)$

=>F(s)=$\frac{f(0^+)}{s}$+$\frac{1}{s}\int_0^{\infty} e^{-st}\left[\frac{df(t)}{dt}\right]dt$

=>F(s)= $\frac{f(0^+)}{s} + \frac{1}{s}\ L\left[\frac{df(t)}{dt}\right]$

=> $L\left[\frac{df(t)}{dt}\right]$ =s F(s) $-f(0^+)$

(iv) Integration by time "t":-

$$L\left[\int_0^\infty f(t)dt\right] = \int_0^\infty \left[\int_0^\infty f(t)dt\right] e^{-st}\text{dt}$$

U=$\int_0^\infty f(t)dt$ => $f(t) = \frac{du}{dt} => du = f(t)dt$

dv = e^{-st}dt => v= $\frac{-e^{-st}}{s}$

So, $L\left[\int_0^\infty f(t)dt\right] = L\int_0^\infty udv = u[v]_0^\infty - \int_0^\infty vdu$

$$= \frac{-e^{-st}}{s}\int_0^\infty \int_\infty^\infty f(t)dt - \frac{1}{s}\int_\infty^\infty f(t)e^{-st}dt$$

$$= \frac{1}{s}\left[\int_\infty^\infty f(t)dt\right]_0^\infty + \frac{\text{F(s)}}{s}$$

(v) . Differentiation w.r.to S [frequency differentiation]:-

$$\frac{dF(s)}{ds} = -L[t.f(t)$$

Proof: $\frac{dF(s)}{ds} = \frac{d}{ds}\int_0^\infty f(t).e^{-st}.dt = \int_0^\infty f(t)\left[\frac{de^{-st}}{ds}\right]dt = \int_0^\infty f(t)e^{-st}(-t)dt$

$$= -\int_0^\infty tf(t).e^{-st}.dt = -L[t.f(t)]$$

(vi) . Integration by 'S':-

$$\int_s^\infty F(s) = L\left[\frac{f(t)}{t}\right]$$

Proof; $\int_s^\infty F(s) = \int_0^\infty \int_0^\infty f(t).e^{-st}.ds.dt = \int_0^\infty f(t)\left[\frac{de^{-st}}{-t}\right]_0^\infty dt$

$=\int_0^\infty f(t)\left[0 - \frac{de^{-st}}{-t}\right]dt = \int_0^\infty \frac{f(t)}{-t}.e^{-st}.dt = L\left[\frac{f(t)}{t}\right]$

(vii). Shifting Theorem:-

(a) L[f(t-1).U(t-a)] = $e^{-as}F(s)$

(b) F(s+a) =L[$e^{-as}f(t)$]

Proof: L [$e^{-as}f(t)$] = $e^{-(a+s)t}f(t).dt = F(s+a)$

(viii). Initial Value Theorem:-

$f(0^+) = \underset{t \to 0}{Lt} f(t) = \underset{s \to \infty}{Lt} [sF(s)$

proof : sF(s) - $f(0^+) = \int_0^\infty \frac{df(t)}{dt}.e^{-st}.dt$

=>s(s) = $f(0^+) + \int_0^\infty \frac{df(t)}{dt}.e^{-st}.dt$

=> $\underset{s \to \infty}{Lt} sf(s) = f(0^+) + \underset{s \to \infty}{Lt} \int_0^\infty \frac{df(t)}{dt}.e^{-st}.dt = f(0^+)$

(ix). Final Value Theorem:-

$F(\infty) = \underset{t \to \infty}{Lt} f(t) = \underset{s \to 0}{Lt} [sf(s)]$

Proof :- $[sf(s) - f(0^+)] = \underset{s \to 0}{Lt} \int_0^\infty \frac{df(t)}{dt}.e^{-st}.dt = \int_0^\infty \frac{df(t)}{dt}.dt = \int_0^\infty df(t).dt = f(t)_0^\infty$

$= f(\infty) - f(0) = f(\infty) = \underset{t \to \infty}{Lt} f(t)$

(x). Theorem of periodic functions:-

Let $f_1(t), f_2(t), f_3(t), \dots \dots be$ the functions described by 1^{st} ,2^{nd} & 3^{rd} ...cycles of the periodic function $f(t)$, whose time periods is T.

$$f(t) = f_1(t) + f_2(t) + f_3(t) + \cdots \dots = f_1(t) + f_1(t-T) + f_1(t-2T)$$

$$L[f(t)] = F_1(s) + e^{-ST}F_1(s) + e^{-2ST}F_1(s) + \cdots$$

$$= F_1(s)\ [1 + e^{-ST} + e^{-2ST} + \cdots] \ = F_1(s)$$

(xi). Convolution Theorem:

$$L^{'}[F_1(s)F_2(s)] = f_1(t) * f_2(t) = \int_0^t f_1(t-\tau) f(\tau) d\tau$$

(xii). Time Scaling:

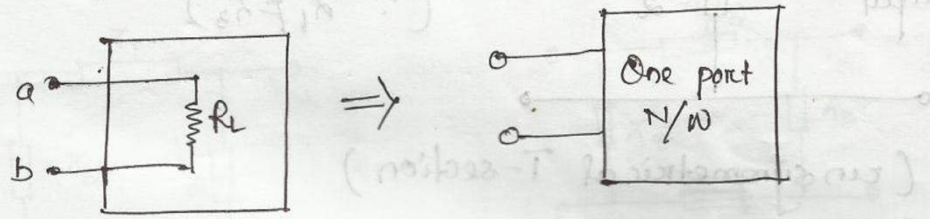
$$L[f(at)] = \frac{1}{a} F\left(\frac{s}{a}\right)$$

Two Port Network Analyse's

Port in Network

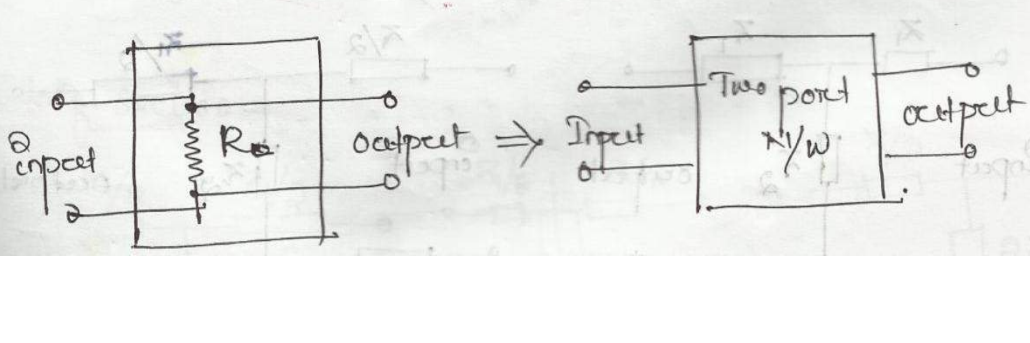
(a) One port Network —

Any ~~network~~ active or passive network having only two terminals can be represented by an one port network.



(b) Two port network —

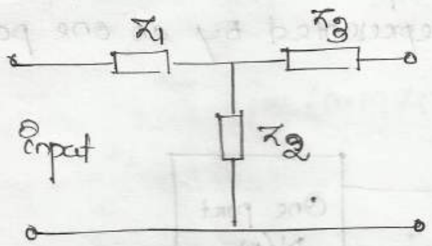
If a network consists of two pairs of terminals (i.e four terminals) where one pair of terminals can be designed as input. & the other pair being output. It is called a two port network (or four terminal N/W).



Network Configuration

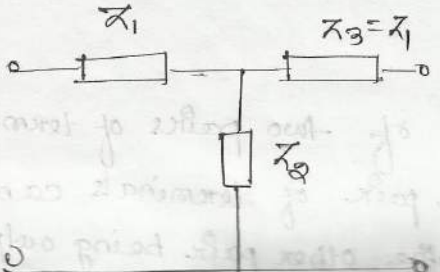
Depending on the configuration of impedance a network can be specified in to following sections —

(i) T-Section —



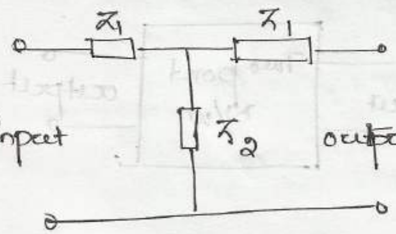
$(\because Z_1 \neq Z_3)$

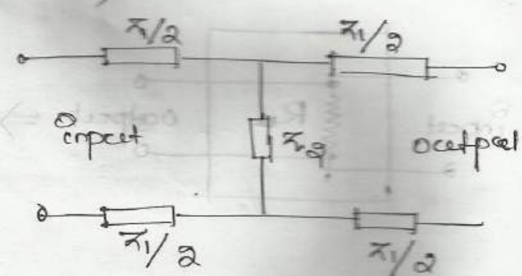
(unsymmetrical T-section)



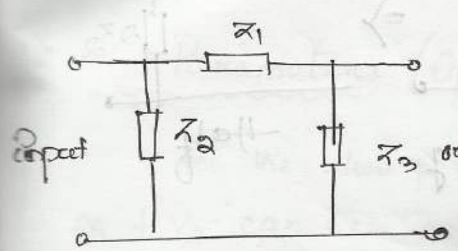
$(Z_1 = Z_3)$

(Symmetrical T-section)

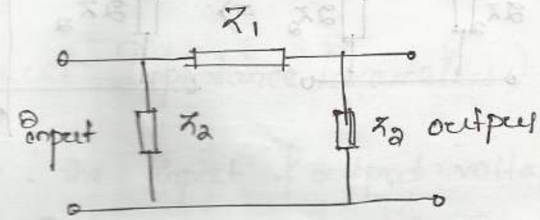




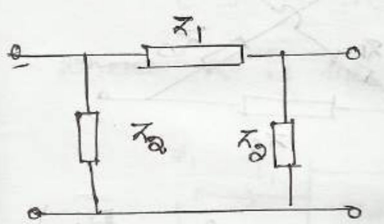
(ii) π-Section –



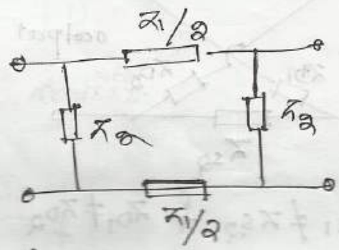
(Asymmetrical π-section)
($Z_2 \neq Z_3$)



(symmetrical π-section)

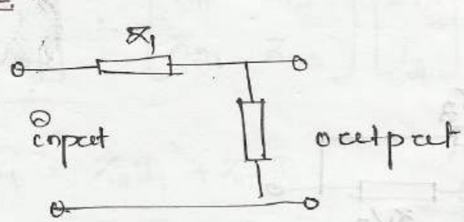


(unbalanced π-section)

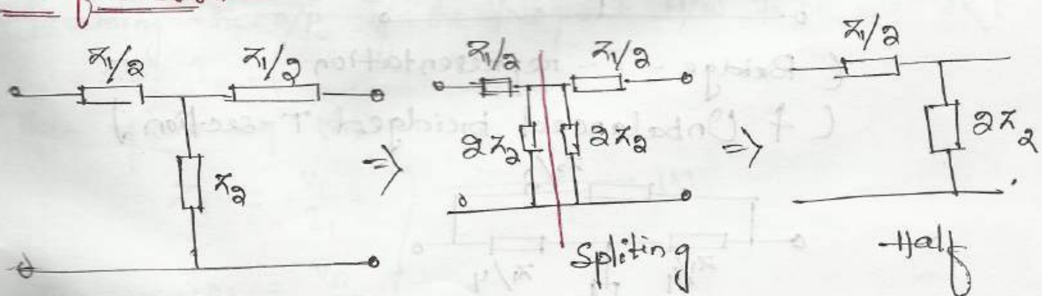


(Balanced π-section)

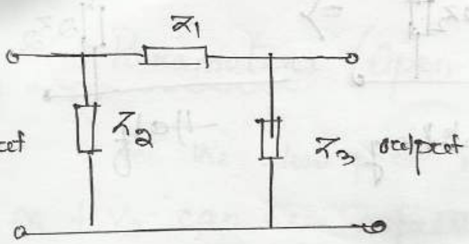
(iii) L-section



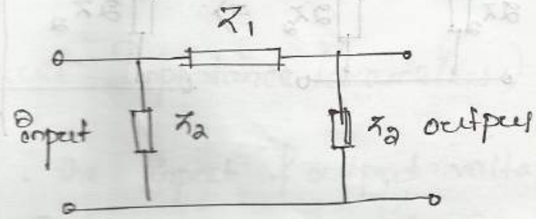
(iv) Half-section –



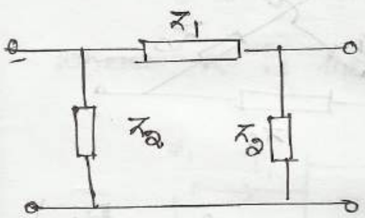
(ii) π-Section –



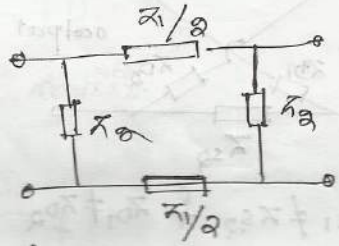
(Asymmetrical π-section)
($Z_2 \neq Z_3$)



(symmetrical π-section)

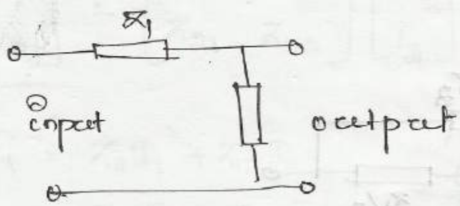


(unbalanced π-section)

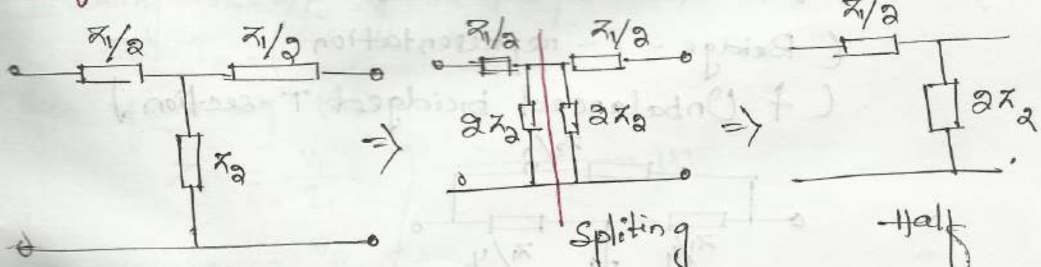


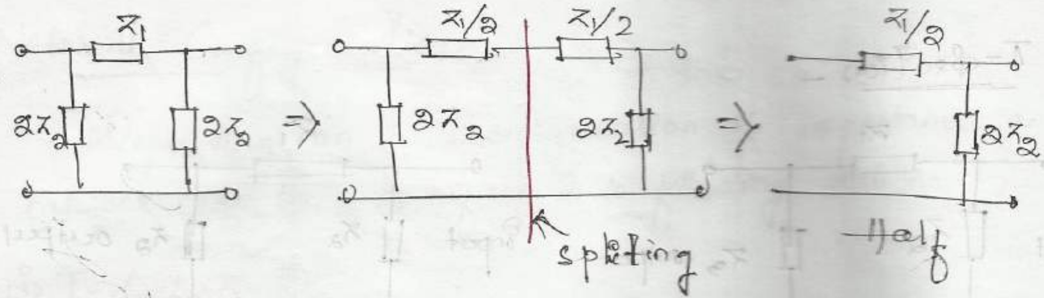
(Balanced π-section)

(iii) L-section

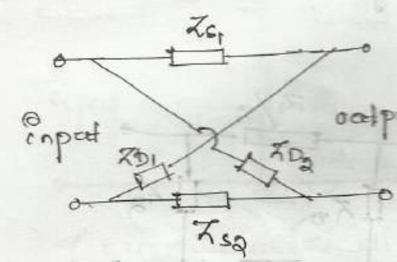


(iv) Half-section –

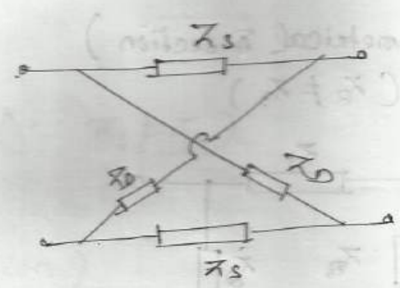




(v) Lattice Section

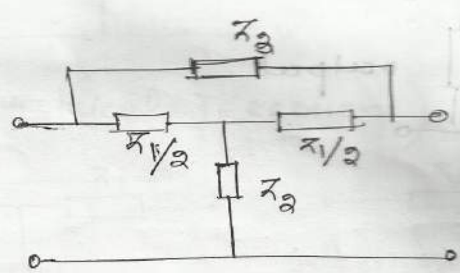


$Z_{S_1} \neq Z_{S_2}$ & $Z_{D_1} \neq Z_{D_2}$
(Asymmetrical section)

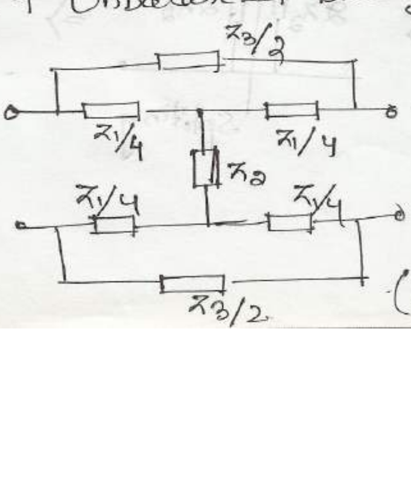


(Symmetrical section)

(VI) Bridge T-section



(Bridge - T - representation)
(& Unbalanced bridged T-section)



(Balanced bridge T-section)

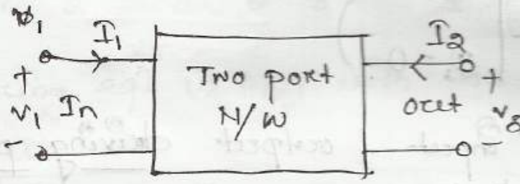
Parameter Representation

1) Z- Parameters (Open Circuit Impedance Parameters)

For the two port N/w, the input & output voltages V_1 & V_2 can be expressed in terms of input & output currents I_1 & I_2 respectively, as

$$[V] = [Z][I] \quad \text{---} (1)$$

where Z is the impedance matrix.



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad \text{---} (2)$$

$$\left.\begin{aligned} V_1 &= Z_{11} I_1 + Z_{12} I_2 \\ V_2 &= Z_{21} I_1 + Z_{22} I_2 \end{aligned}\right\} \quad \text{---} (3)$$

Assuming the o/p of the two port N/w to be open ckt
i.e. $I_2 = 0$

Then from eq^n (3)

$$\left.\begin{aligned} Z_{11} &= \frac{V_1}{I_1} \\ Z_{21} &= \frac{V_2}{I_1} \end{aligned}\right\} \quad \text{---} (4)$$

Again, assuming the i/p port of the same two port N/w to be open circuited.

i.e $I_1 = 0$

Then

$$\left.\begin{aligned} Z_{12} &= \frac{V_1}{I_2} \\ Z_{22} &= \frac{V_2}{I_2} \end{aligned}\right\} \quad -(5)$$

So, $Z_{11} = \left(\frac{V_1}{I_1} \bigg|_{I_2 = 0} \right)$

& $Z_{22} = \left(\frac{V_2}{I_2} \bigg|_{I_1 = 0} \right)$

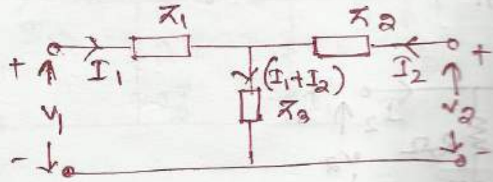
are called input & output driving point impedances respectively.

while $Z_{12} = \left(\frac{V_1}{I_2} \bigg|_{I_1 = 0} \right)$

& $Z_{21} = \left(\frac{V_2}{I_1} \bigg|_{I_2 = 0} \right)$

are called reverse & forward transfer impedances.

Ex-1 :- Find the Z-parameters for the following network.



Sol^n

$$V_1 - I_1 Z_1 - (I_1+I_2) Z_3 = 0$$

$$\Rightarrow V_1 = I_1(Z_1 + Z_3) + I_2 Z_3 \quad \text{---}(1)$$

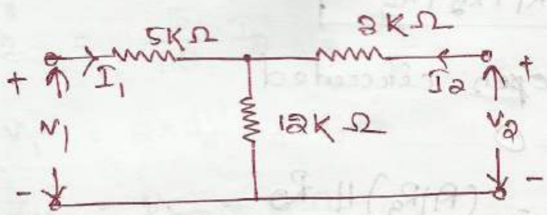
$$V_2 - I_2 Z_2 - (I_1+I_2) Z_3 = 0$$

$$\Rightarrow V_2 = I_2(Z_2 + Z_3) + I_1 Z_3 \quad \text{---}(2)$$

Comparing eq^{n}s (1) & (2) with the standard Z-parameter equations

$Z_{11} = Z_1 + Z_3$,	$Z_{12} = Z_3$
$Z_{21} = Z_3$	$Z_{22} = Z_2 + Z_3$

Ex-2



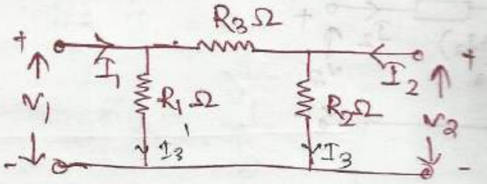
$Z_{11} = 5+12 = 17\ K\Omega$

$Z_{12} = 12K\Omega$

$Z_{21} = 12K\Omega$

$Z_{22} = 3+12 = 15K\Omega$

Ex-3 Determine the z-parameters for a π-type attenuator section.



soln Let o/p is open circuited.

i.e $I_2 = 0$

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$$

$$= (R_2+R_3) \parallel R_1 = \frac{R_1(R_2+R_3)}{R_1+R_2+R_3}$$

$$I_3 = \frac{R_1}{R_1+R_2+R_3} I_1$$

$$V_2 = I_3 R_2 = I_1 \frac{R_1 R_2}{R_1+R_2+R_3}$$

$$Z_{21} = \frac{V_2}{I_1} = \frac{R_1 R_2}{R_1+R_2+R_3}$$

Again i/p is open circuited

i.e $I_1 = 0$

$$Z_{22} = \frac{V_2}{I_2} = (R_1+R_3) \parallel R_2$$

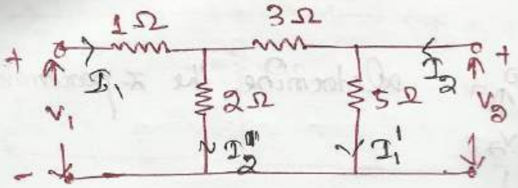
$$Z_{22} = \frac{R_2(R_1+R_3)}{R_1+R_2+R_3}$$

$$I_3' = \frac{R_2 I_2}{R_1+R_2+R_3}$$

$$V_1 = I_3' R_1 = \frac{R_1 R_2 I_2}{R_1+R_2+R_3}$$

$$\Rightarrow Z_{12} = \frac{V_1}{I_2} = \frac{R_1 R_2}{R_1+R_2+R_3}$$

Ex-4



Find the z-parameters.

sol^n

Case-1 $(I_2 = 0)$

$$z_{11} = (8 \parallel 2) + 1 = \frac{16}{10} + 1 = \frac{16+10}{10} = 2.6\,\Omega$$

$$I_1' = \frac{2}{10} \times I_1 = \frac{I_1}{5}$$

$$V_2 = 5 I_1' = 5 \times \frac{I_1}{5} = I_1$$

$$\Rightarrow z_{21} = \frac{V_2}{I_1} = 1\,\Omega$$

Case-2 $(I_1 = 0)$

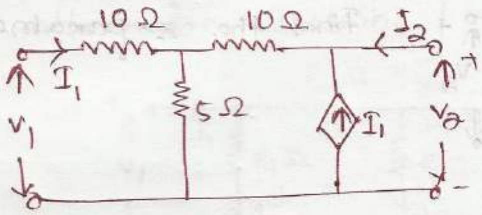
$$z_{22} = 5 \parallel 5 = 2.5\,\Omega$$

$$I_2'' = \frac{5}{10} I_2 = \frac{I_2}{2}$$

$$V_1 = 2 \times I_2' = I_2$$

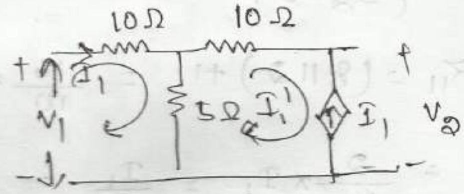
$$\Rightarrow z_{12} = \frac{V_1}{I_2} = 1\,\Omega$$

Ex-5



Determine the z-parameters.

Sol^n



$V_1 - 10I_1 - 5(I_1 - I_1') = 0$

$\Rightarrow V_1 = 15I_1 - 5I_1'$

From the ckt , $I_1 = -I_1'$

So, $V_1 = 20I_1$

$\Rightarrow \frac{V_1}{I_1} = 20 \Rightarrow \boxed{Z_{11} = 20\,\Omega}$

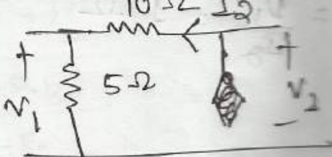
$V_2 + 10I_1' - 5(I_1 - I_1') = 0$

$\Rightarrow V_2 = 10I_1 + 5I_1 - 5I_1' = 10I_1 + 5I_1 + 5I_1$

$\Rightarrow V_2 = 20I_1$

$\Rightarrow \frac{V_2}{I_1} = 20 \Rightarrow \boxed{Z_{21} = 20\,\Omega}$

Case-2

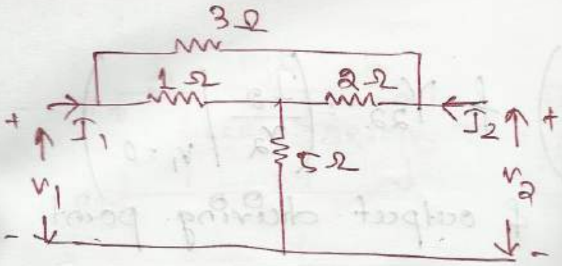


$\boxed{Z_{22} = 15\,\Omega}$

$V_1 = 5I_2$

$\Rightarrow \frac{V_1}{I_2} = 5$

$\Rightarrow \boxed{Z_{12} = 5\,\Omega}$



Find z-parameters.

Ans $z_{11} = \frac{35}{6}\Omega$ $z_{12} = \frac{16}{3}\Omega$

$z_{21} = \frac{16}{3}\Omega$, $z_{22} = \frac{19}{3}\Omega$

2) Y-Parameters (Short-Circuit Admittance Parameters)

In a two port network, the input currents I_1 & I_2 can be expressed in terms of input & output voltages V_1 & V_2 respectively as

$$[I] = [Y][V] \quad \text{——— (1)}$$

where $[Y]$ is the admittance matrix.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\left.\begin{aligned} I_1 &= Y_{11}V_1 + Y_{12}V_2 \\ I_2 &= Y_{21}V_1 + Y_{22}V_2 \end{aligned}\right\} \quad \text{——— (2)}$$

Assuming the output of the two port n/w to be short-circuited,

i.e $V_2 = 0$

$$Y_{11} = \frac{I_1}{V_1} \quad \& \quad Y_{21} = \frac{I_2}{V_1}$$

Similarly, assuming the input of the two port n/w to be short circuited, i.e $V_1 = 0$.

$$Y_{12} = \frac{I_1}{V_2} \quad \& \quad Y_{22} = \frac{I_2}{V_2}$$

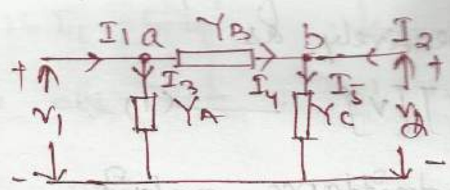
Here, $\left(Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}\right)$ & $Y_{22} = \left(\left.\frac{I_2}{V_2}\right|_{V_1=0}\right)$

are called input & output driving point admittances.

$$Y_{12} = \left(\left.\frac{I_1}{V_2}\right|_{V_1=0}\right) \text{ \& } Y_{21} = \left(\left.\frac{I_2}{V_1}\right|_{V_2=0}\right)$$

are called reverse & forward transfer admittances.

Ex-1



Find the Y-parameters.

Sol^n

At node-'a'

$$I_1 = I_3 + I_4$$

$$= V_1 Y_A + (V_1 - V_2) Y_B$$

$$I_1 = V_1 (Y_A + Y_B) + (-Y_B) V_2 \quad \text{—— (1)}$$

At node-'b'

$$I_2 + I_4 = I_5$$

$$\Rightarrow I_2 = I_5 - I_4$$

$$= V_2 Y_C - (V_1 - V_2) Y_B$$

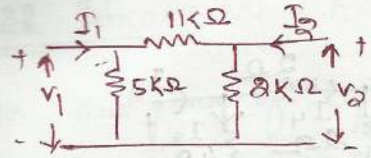
$$\Rightarrow I_2 = (-Y_B) V_1 + (Y_C + Y_B) V_2 \quad \text{—— (2)}$$

Comparing eq^n (1) & (2) with the standard eq^ns of Y-parameter.

$$Y_{11} = Y_A + Y_B, \quad Y_{12} = -Y_B$$

$$Y_{21} = -Y_B, \quad Y_{22} = Y_C + Y_B$$

Ex-2



Find the Y-parameters of the π N/W.

Sol^n

$$Y_A = \frac{1}{5\,K\Omega} = .2\times10^{-3}\ \mho$$

$$Y_B = \frac{1}{1\,K\Omega} = 10^{-3}\ \mho$$

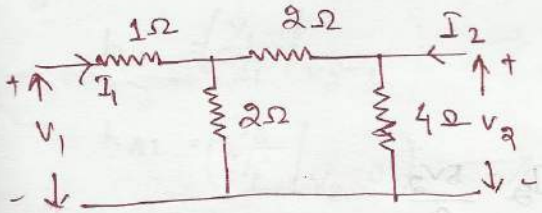
$$Y_C = \frac{1}{2\,K\Omega} = .5\times10^{-3}\ \mho$$

$$Y_{11} = Y_A + Y_B = 1.2\times10^{-3}\ \mho$$

$$Y_{12} = Y_{21} = -Y_B = -10^{-3}\ \mho$$

$$Y_{22} = Y_C + Y_B = 1.5\times10^{-3}\ \mho$$

Ex-3



find Y-parameters of the N/W.

Sol^n

case-1 $(V_2 = 0)$

$$Y_{11} = \frac{I_1}{V_1} = \frac{1}{(2\|2)+1}$$

$$\boxed{Y_{11} = \frac{1}{2}\ \mho}$$

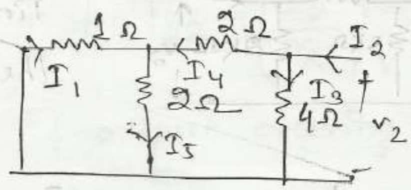
$$Y_{21} = \frac{I_2}{V_1}$$

$$I_2 = -\left(\frac{2}{4}\right)I_1 = -I_1/2$$

$$I_1 = \frac{V_1}{2}$$

$$\Rightarrow -2I_2 = V_1/2 \Rightarrow \boxed{Y_{21} = \frac{I_2}{V_1} = -\frac{1}{4}\ \mho}$$

Case-2 $V_1 = 0$



$$Y_{22} = \frac{1}{[(1 \parallel 2) + 2] \parallel 4}$$

$$= \frac{1}{\frac{\left(\frac{2}{3}+2\right)\times 4}{\frac{2}{3}+2+4}} = \frac{\frac{2}{3}+6}{\frac{8}{3}\times 4}$$

$$= \frac{20}{32} = \frac{5}{8}$$

$$\Rightarrow \boxed{Y_{22} = \frac{5}{8}\ \mho}$$

$$Y_{12} = \frac{I_1}{V_2}$$

$$I_2 = I_3 + I_4$$

$$\Rightarrow I_4 = I_2 - I_3$$

$$I_2 = V_2 \times \frac{5}{8} \Rightarrow I_2 = \frac{5V_2}{8}$$

So, $$I_4 = \frac{5V_2}{8} - \frac{V_2}{4} = \frac{3V_2}{8}$$

$$I_1 = -\frac{2}{3} \times \frac{3V_2}{8} = -\frac{V_2}{4}$$

$$\Rightarrow \boxed{Y_{12} = \frac{I_1}{V_2} = -\frac{1}{4}\ \mho}$$

Hybrid Parameters (h-parameter)

In this form of representation, the voltage of the input port & the current of the output port are expressed in terms of the current of the input port & the voltage of the output port.

i.e $$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \quad \text{——— (1)}$$

$$\left.\begin{aligned} V_1 &= h_{11} I_1 + h_{12} V_2 \\ I_2 &= h_{21} I_1 + h_{22} V_2 \end{aligned}\right\} \text{——— (2)}$$

Assuming short circuit conditions at the output
i.e $V_2 = 0$

$$h_{11} = \left(\frac{V_1}{I_1} \Big|_{V_2 = 0} \right) \rightarrow \text{input impedance}$$

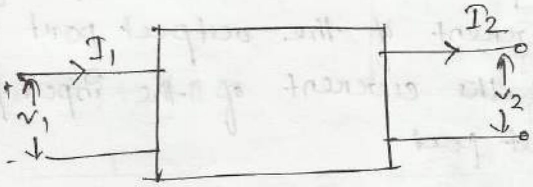
$$h_{21} = \left(\frac{I_2}{I_1} \Big|_{V_2 = 0} \right) \rightarrow \text{forward current gain}$$

Again, for the input open-circuited condition,
i.e $I_1 = 0$

$$h_{12} = \left(\frac{V_1}{V_2} \Big|_{I_1 = 0} \right) \rightarrow \text{Reverse voltage gain}$$

$$h_{22} = \left(\frac{I_2}{V_2} \Big|_{I_1 = 0} \right) \rightarrow \text{output admittance}$$

ABCD Parameters



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$V_1 = AV_2 + B(-I_2)$$
$$I_1 = CV_2 + D(-I_2)$$

Assuming the receiving end to be open circuited

i.e $I_2 = 0$

$$A = \left(\frac{V_1}{V_2} \Big|_{I_2=0} \right) \rightarrow \text{Reverse voltage ratio}$$

$$C = \left(\frac{I_1}{V_2} \Big|_{I_2=0} \right) \rightarrow \text{Transfer admittance}$$

Assuming, receiving end to short circuited

i.e $V_2 = 0$

$$B = \left(-\frac{V_1}{I_2} \Big|_{V_2=0} \right) \rightarrow \text{Transfer impedance}$$

$$D = \left(-\frac{I_1}{I_2} \Big|_{V_2=0} \right) \rightarrow \text{Reverse current ratio}$$

Inter - Relationship between Parameters of two port Network

1) Z - parameters in terms of Y - parameters

$$[Z] = [Y]^{-1}$$

$\Rightarrow$ $\begin{matrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{matrix}$

$$Z_{11} = \frac{Y_{22}}{\Delta Y}, \quad Z_{12} = -\frac{Y_{12}}{\Delta Y}$$

$$Z_{21} = -\frac{Y_{21}}{\Delta Y}, \quad Z_{22} = \frac{Y_{11}}{\Delta Y}$$

where $\Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21}$

2) Z- parameters in terms of ABCD parameters

$$Z_{11} = \frac{A}{C}, \quad Z_{12} = \frac{AD - BC}{C}$$

$$Z_{21} = \frac{1}{C}, \quad Z_{22} = \frac{D}{C}$$

3) Z- parameters in terms of h-parameters

$$Z_{11} = \frac{\Delta h}{h_{22}}, \quad Z_{12} = \frac{h_{12}}{h_{22}}$$

$$Z_{21} = -\frac{h_{21}}{h_{22}}, \quad Z_{22} = \frac{1}{h_{22}}$$

where $\Delta h = h_{11}h_{22} - h_{12}h_{21}$

4) Y-parameters in terms of Z-parameter

$$Y_{11} = \frac{Z_{22}}{\Delta Z}, \quad Y_{12} = -\frac{Z_{12}}{\Delta Z}$$

$$Y_{21} = -\frac{Z_{21}}{\Delta Z}, \quad Y_{22} = \frac{Z_{11}}{\Delta Z}$$

5) Y-parameters in terms of ABCD parameters

$$Y_{11} = \frac{D}{B}, \quad Y_{12} = -\frac{AD - BC}{B}$$

$$Y_{21} = -\frac{1}{B}, \quad Y_{22} = \frac{A}{B}$$

6) h-parameters in terms of Z-parameters

$$h_{11} = \frac{\Delta Z}{Z_{22}}, \quad h_{12} = \frac{Z_{12}}{Z_{22}}$$

$$h_{21} = -\frac{Z_{21}}{Z_{22}}, \quad h_{22} = \frac{1}{Z_{22}}$$

7) h-parameters in terms of Y-parameters

$$h_{11} = \frac{1}{Y_{11}}, \quad h_{12} = -\frac{Y_{12}}{Y_{11}}$$

$$h_{21} = \frac{Y_{21}}{Y_{11}}, \quad h_{22} = \frac{\Delta Y}{Y_{11}}$$

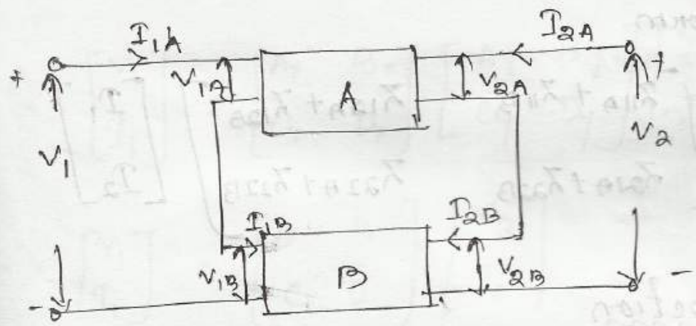
8) h-parameters in terms of ABCD parameters

$$h_{11} = \frac{B}{D} \quad h_{12} = \left(\frac{AD - BC}{D}\right)$$

$$h_{21} = -\left(\frac{1}{D}\right) \quad h_{22} = \left(\frac{C}{D}\right)$$

Different types of Interconnections of two port n/w

1) Series Connection



for N/W A

$$V_{1A} = Z_{11A} I_{1A} + Z_{12A} I_{2A}$$

$$V_{2A} = Z_{21A} I_{1A} + Z_{22A} I_{2A}$$

for N/W B

$$V_{1B} = Z_{11B} I_{1B} + Z_{12B} I_{2B}$$

$$V_{2B} = Z_{21B} I_{1B} + Z_{22B} I_{2B}$$

From, the circuit

$$I_1 = I_{1A} = I_{1B}$$

$$I_2 = I_{2A} = I_{2B}$$

$$V_1 = V_{1A} + V_{2B}$$

$$V_2 = V_{2A} + V_{2B}$$

$$v_1 = v_{1A} + v_{1B}$$

$$= (z_{11A} I_{1A} + z_{12A} I_{2A}) + (z_{11B} I_{1B} + z_{12B} I_{2B})$$

$$= I_1 (z_{11A} + z_{11B}) + I_2 (z_{12A} + z_{12B})$$

Simillarly $v_2 = v_{2A} + v_{2B}$

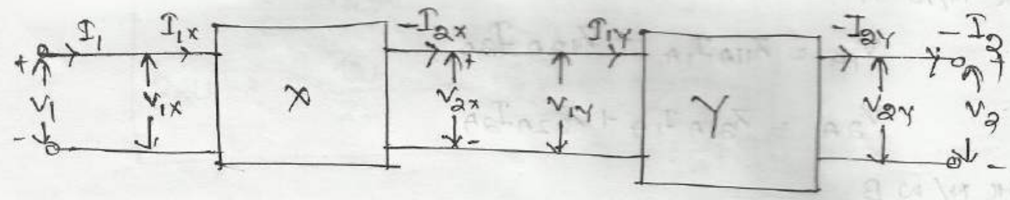
$$= (z_{21A} I_{1A} + z_{22A} I_{2A}) + (z_{21B} I_{1B} + z_{22B} I_{2B})$$

$$= I_1 (z_{21A} + z_{21B}) + I_2 (z_{22A} + z_{22B})$$

In matrix form

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} z_{11A} + z_{11B} & z_{12A} + z_{12B} \\ z_{21A} + z_{22B} & z_{22A} + z_{22B} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

2) Cascade Connection



For N/W 'X'

$$V_{1X} = A_X V_{2X} - B_X I_{2X}$$

$$I_{1X} = C_X V_{2X} - D_X I_{2X}$$

For N/W 'Y'

$$V_{1Y} = A_Y V_{2Y} - B_Y I_{2Y}$$

$$I_{1Y} = C_Y V_{2Y} - D_Y I_{2Y}$$

For the cascade connection

$$I_1 = I_{1x} \quad , \quad -I_{2x} = I_{1y} \quad , \quad I_2 = I_{2y}$$

$$V_1 = V_{1x} \quad , \quad V_{2x} = V_{1y} \quad , \quad V_2 = V_{2y}$$

$$\begin{bmatrix} V_{1x} \\ I_{1x} \end{bmatrix} = \begin{bmatrix} A_x & B_x \\ C_x & D_x \end{bmatrix} \begin{bmatrix} V_{2x} \\ -I_{2x} \end{bmatrix} = \begin{bmatrix} A_x & B_x \\ C_x & D_x \end{bmatrix} \begin{bmatrix} V_{1y} \\ I_{1y} \end{bmatrix}$$

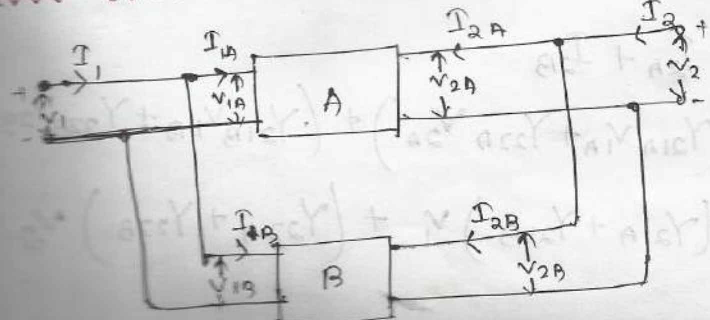
$$\Rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_x & B_x \\ C_x & D_x \end{bmatrix} \begin{bmatrix} A_y & B_y \\ C_y & D_y \end{bmatrix} \begin{bmatrix} V_{2y} \\ -I_{2y} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_x & B_x \\ C_x & D_x \end{bmatrix} \begin{bmatrix} A_y & B_y \\ C_y & D_y \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

where $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_x & B_x \\ C_x & D_x \end{bmatrix} \begin{bmatrix} A_y & B_y \\ C_y & D_y \end{bmatrix}$

3) Parallel Connection



For N/W 'A'

$$I_{1A} = Y_{11A} V_{1A} + Y_{12A} V_{2A}$$

$$I_{2A} = Y_{21A} V_{1A} + Y_{22A} V_{2A}$$

For N/W B

$$I_{1B} = Y_{11B} V_{1B} + Y_{12B} V_{2B}$$

$$I_{2B} = Y_{21B} V_{1B} + Y_{22B} V_{2B}$$

For parallel connection

$$V_1 = V_{1A} = V_{1B}$$

$$V_2 = V_{2A} = V_{2B}$$

$$I_1 = I_{1A} + I_{1B}$$

$$I_2 = I_{2A} + I_{2B}$$

So, $I_1 = I_{1A} + I_{1B}$

$$= (Y_{11A} V_{1A} + Y_{12A} V_{2A}) + (Y_{11B} V_{1B} + Y_{12B} V_{2B})$$

$$= (Y_{11A} + Y_{11B}) V_1 + (Y_{12A} + Y_{12B}) V_2$$

Simillarly

$$I_2 = I_{2A} + I_{2B}$$

$$= (Y_{21A} V_{1A} + Y_{22A} V_{2A}) + (Y_{21B} V_{1B} + Y_{22B} V_{2B})$$

$$= (Y_{21A} + Y_{21B}) V_1 + (Y_{22A} + Y_{22B}) V_2$$

In matrix form

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11A} + Y_{11B} & Y_{12A} + Y_{12B} \\ Y_{21A} + Y_{21B} & Y_{22A} + Y_{22B} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

MODULE- III

Fourier Analysis

Fourier Series

Any arbitrary periodic function can be represented by an infinite series of sinusoidals of harmonically related frequencies. This infinite series is known as Fourier Series.

If $f(t)$ is a periodic function, then the Fourier series is

$$f(t) = a_0 + a_1 \cos \omega_0 t + a_2 \cos 2\omega_0 t + \cdots + a_n \cos n\omega_0 t$$
$$+ b_1 \sin \omega_0 t + b_2 \sin 2\omega_0 t + \cdots + b_n \cos n\omega_0 t$$
$$= a_0 + \sum_{n=1}^{\infty} \left(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t \right)$$

where $\omega_0 = \frac{2\pi}{T_0}$

- is the fundamental frequency.

$n\omega_0$ is the n^{th} harmonic of fundamental frequency

a_0, a_n, b_n are the Fourier Co-efficients.

where

$$a_0 = \frac{1}{T_0} \int_0^{T_0} f(t)\, dt$$

$$a_n = \frac{2}{T_0} \int_0^{T_0} f(t) \cos n\omega_0 t\, dt$$

$$b_n = \frac{2}{T_0} \int_0^{T_0} f(t) \sin n\omega_0 t\, dt$$

Dirichlet's Condition

The conditions, under which a periodic function $f(t)$ can be expanded in a convergent Fourier series are known as Dirichlet's conditions.

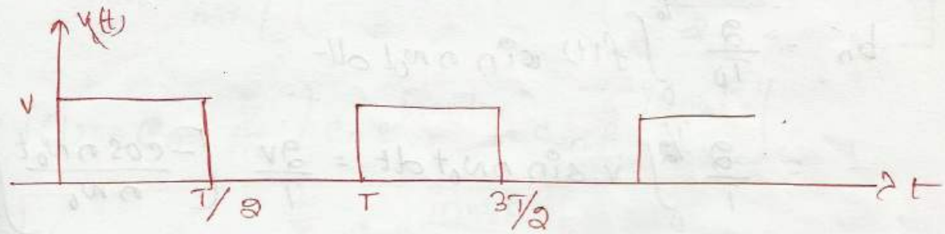
(i) $f(t)$ is a single valued function

(ii) $f(t)$ has a finite number of discontinuities in each period T.

(iii) $f(t)$ has a finite no. of maxima & minimas in each period T.

(iv) The integral $\int_0^T |f(t)|\,dt$ exists & is finite or in other way $\int_0^T |f(t)|^2 dt < \infty$.

Ex



Find the Fourier series expansion of the periodic wave form.

Sol

$v(t) = V$ for $0 \leq t < T/2$

$\quad\; 0$ for $T/2 < t < T$

$a_0 = \frac{1}{T_0}\int_0^{T_0} f(t)\,dt = \frac{1}{T}\int_0^{T/2} V\,dt = \frac{V}{T} \times \frac{T}{2}$

$\Rightarrow \boxed{a_0 = \frac{V}{2}}$

$$a_n = \frac{2}{T_0}\int_0^{T_0} f(t)\cos n w_0 t \, dt$$

$$= \frac{2}{T}\int_0^{T/2} V \cos n\left(\frac{2\pi}{T}\right) t \, dt$$

$$= \frac{2V}{T}\left[\frac{\sin n w_0 t}{n w_0}\right]_0^{T/2}$$

$$= \frac{2V}{T \times n w_0} \sin\left(n \frac{2\pi}{T} \times \frac{T}{2}\right)$$

$$= \frac{2V}{T} \times \frac{T}{n \times 2\pi} \sin n\pi$$

$$= \frac{2V}{2n\pi} \sin n\pi$$

$$\Rightarrow \boxed{a_n = 0}$$

$$b_n = \frac{2}{T_0}\int_0^{T_0} f(t) \sin n w_0 t \, dt$$

$$= \frac{2}{T}\int_0^{T/2} V \sin n w_0 t \, dt = \frac{2V}{T}\left[-\frac{\cos n w_0 t}{n w_0}\right]_0^{T/2}$$

$$= \frac{-2V}{T \times n w_0}\left(\cos n \times \frac{2\pi}{T} \times \frac{T}{2} - 1\right)$$

$$= \frac{-2V \times T}{T \times n \times 2\pi}\left(\cos n\pi - 1\right) = \frac{V}{n\pi}\left(1 - \cos n\pi\right)$$

$$= \frac{V}{n\pi} \Big[$$

$= 0$ for even n

$= \frac{2V}{n\pi}$ for odd n

So, the F.S of the square wave is

$$V(t) = V\left[\frac{1}{2} + \frac{2}{\pi}\sin n\omega_0 t + \frac{2}{3\pi}\sin 3\omega_0 t + \frac{2}{5\pi}\sin 5\omega_0 t + \cdots\right]$$

Exponential form of Fourier Series

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t\right)$$

$$= a_0 + \sum_{n=1}^{\infty}\left[\frac{a_n\left(e^{jn\omega_0 t} + e^{-jn\omega_0 t}\right)}{2} + \frac{b_n\left(e^{jn\omega_0 t} - e^{-jn\omega_0 t}\right)}{2j}\right]$$

$$= a_0 + \sum_{n=1}^{\infty}\frac{1}{2}\left[\left(a_n + \frac{b_n}{j}\right)e^{jn\omega_0 t} + \left(a_n - \frac{b_n}{j}\right)e^{-jn\omega_0 t}\right]$$

$$= a_0 + \sum_{n=1}^{\infty}\left[\left(\frac{a_n - jb_n}{2}\right)e^{jn\omega_0 t} + \left(\frac{a_n + jb_n}{2}\right)e^{-jn\omega_0 t}\right]$$

$$\Rightarrow f(t) = C_0 + \sum_{n=1}^{\infty}\left[c_n e^{jn\omega_0 t} + c_{-n} e^{-jn\omega_0 t}\right]$$

$$\Rightarrow \boxed{f(t) = C_0 + \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}}$$

Where

$$C_0 = a_0$$

$$c_n = \frac{1}{T_0}\int_0^{T_0} f(t)\, e^{-jn\omega_0 t}\, dt$$

Trigonometric Fourier Series

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t \right)$$

$$= A_0 + \sum_{n=1}^{\infty} A_n \cos (n\omega_0 t - \phi_n)$$

Where,

$$A_0 = a_0$$

$$A_n = \sqrt{a_n^2 + b_n^2}$$

$$\phi_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)$$

A_n & ϕ_n are called the amplitude & the phase of the n^{th} harmonic respectively.

Variation of A_n with n (or $n\omega_0$) is known as Amplitude spectrum.

Variation of ϕ_n with n (or $n\omega_0$) is known as Phase - spectrum.

Effective Value of a Periodic Function

The effective (or R.M.S) value of a periodic function $f(t)$ is defined as

$$F_{eff}\,(F_{rms}) = \sqrt{\frac{1}{T_0}\int_0^{T_0} [f(t)]^2 dt}$$

$$= \sqrt{\frac{1}{T_0}\int_0^{T_0} \left[A_0 + \sum_{n=1}^{\infty} A_n \cos (n\omega_0 t - \phi_n) \right]^2 dt}$$

$$= \sqrt{\frac{1}{T_0}\left[A_0^2 T_0 + \sum_{n=1}^{\infty} A_n^2 \frac{T_0}{2} \right]}$$

$$\Rightarrow F_{eff}\,(F_{rms}) = \sqrt{A_0^2 + \sum_{n=1}^{\infty} \left(\frac{A_n}{\sqrt{2}}\right)^2}$$

Waveform Symmetry

$$a_0 = \frac{1}{T_0}\int_0^{T_0} f(t)\,dt = \frac{1}{T}\left[\int_{-T_0/2}^{0} f(t)\,dt + \int_0^{T_0/2} f(t)\,dt\right]$$

Putting $t=-x$ in the first integrand & $t=x$ in the 2nd integral

$$a_0 = \frac{1}{T_0}\int_0^{T_0/2}\left[f(x)+f(-x)\right]dx \quad \text{—— (1)}$$

Now

$$a_n = \frac{2}{T_0}\int_0^{T_0} f(t)\cos n\omega_0 t\,dt$$

$$= \frac{2}{T_0}\left[\int_0^{T_0/2} f(t)\cos n\omega_0 t\,dt + \int_{-T_0/2}^{0} f(t)\cos n\omega_0 t\,dt\right]$$

$$= \frac{2}{T_0}\left[\int_0^{T_0/2} f(x)\cos n\omega_0 x\,dx - \int_0^{T_0/2} f(-x)\cos n\omega_0 x(d x)\right]$$

$$a_n = \frac{2}{T_0}\int_0^{T_0/2}\left[f(x)+f(-x)\right]\cos n\omega_0 x\,dx$$

Similarly

$$b_n = \frac{2}{T_0}\int_0^{T_0/2}\left[f(x)-f(x)\right]\sin n\omega x\,dx$$

Types

1. Odd or Rotation symmetry
2. Even or Mirror symmetry
3. Half-wave or alternation symmetry
4. Quarter-wave symmetry.

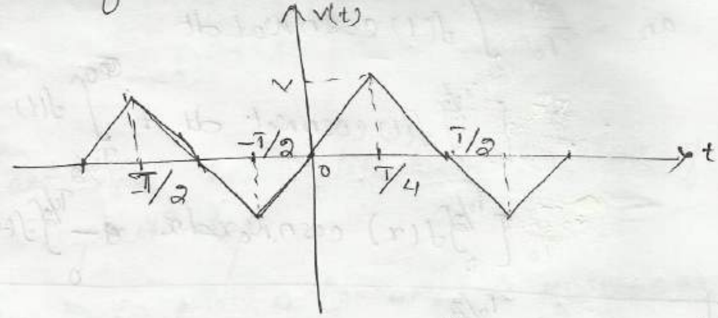
1) Odd Symmetry

A function $f(x)$ is said to be odd if

$$f(x) = f(-x)$$

Hence for odd function $a_0 = 0$ & $a_n = 0$

& $$b_n = \frac{4}{T}\int_0^{T/2} f(x)\sin n\omega x \, dx$$



Thus, the fourier series expansion of an odd function contains only the sine terms, the constant and the cosine terms being zero.

2. Even Symmetry -

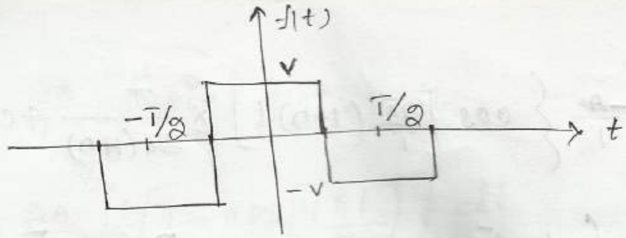
A function $f(x)$ is said to be even if

$$f(x) = f(-x)$$

$$a_0 = \frac{2}{T_0}\int_0^{T_0/2} f(x)\, dx$$

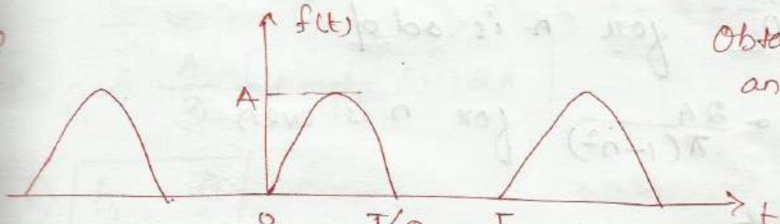
$$a_n = \frac{4}{T_0}\int_0^{T_0/2} f(x)\cos n\omega x \, dx$$

& $$b_n = 0$$



Thus, the Fourier series expansion of an even periodic function contains only the cosine terms plus constant, all sine terms being zero.

Ex-2 Obtain the fourier analysis.



$$f(t) = A \sin\left(\frac{2\pi t}{T}\right), \text{ when } 0 \le t \le T/2$$

$$= 0, \text{ when } T/2 \le t \le T$$

Sol^n

$$a_0 = \frac{1}{T}\int_0^{T/2} A \sin\left(\frac{2\pi t}{T}\right) dt$$

$$= \frac{-A}{T}\left[\frac{\cos\left(\frac{2\pi t}{T}\right)}{\frac{2\pi}{T}}\right]_0^{T/2} = \frac{-A}{T} \times \frac{T}{2\pi}\left[\cos\frac{2\pi t}{T}\right]_0^{T/2}$$

$$= \frac{-A}{2\pi}\left(\cos\pi - 1\right) = \frac{A}{\pi}$$

$$a_n = \frac{2}{T}\int_0^{T/2} A \sin\left(\frac{2\pi t}{T}\right) \cos n\omega_0 t\, dt$$

$$= \frac{2A}{T}\int_0^{T/2} \sin\left(\frac{2\pi t}{T}\right)\cos\left(n\frac{2\pi}{T}t\right) dt$$

$$= \frac{2A}{T}\int_0^{T/2} \frac{1}{2}\left[\sin\frac{2\pi}{T}(1+n)t + \sin\frac{2\pi}{T}(1-n)t\right] dt$$

$$\Rightarrow a_n = \frac{-A}{T}\left\{\cos\left[\frac{2\pi}{T}(1+n)t\right] \times \frac{T}{2\pi(1+n)} + \cos\left[\frac{2\pi}{T}(1-n)t\right] \times \frac{T}{2\pi(1-n)}\right\}_0^{T/2}$$

$$= \frac{-A}{2\pi}\left\{\frac{1}{1+n}\left[\cos\frac{2\pi}{T}(1+n)\frac{T}{2} - 1\right] + \frac{1}{1-n}\left[\cos\frac{2\pi}{T}(1-n)\frac{T}{2} - 1\right]\right\}$$

$$= \frac{-A}{2\pi}\left\{\frac{1}{1+n}\left[\cos(1+n)\pi - 1\right] + \frac{1}{1-n}\left[\cos(1-n)\pi - 1\right]\right\}$$

$\Rightarrow a_n = 0$ for n is odd

$= \frac{2A}{\pi(1-n^2)}$ for n is even

$$b_n = \frac{2}{T}\int_0^{T/2} A\sin\left(\frac{2\pi t}{T}\right)\sin\left(\frac{2\pi n t}{T}\right)dt$$

$$= \frac{2A}{T}\int_0^{T/2}\frac{1}{2}\left\{\cos\left[\frac{2\pi}{T}(1-n)t\right] - \cos\left[\frac{2\pi}{T}(1+n)t\right]\right\}dt$$

$$= \frac{A}{T}\left\{\sin\frac{2\pi}{T}(1-n)t \times \frac{T}{2\pi(1-n)} - \sin\frac{2\pi}{T}(1+n)t \times \frac{T}{2\pi(1+n)}\right\}_0^{T/2}$$

$$= \frac{A}{2\pi}\left\{\frac{1}{(1-n)}\sin\frac{2\pi}{T}(1-n)\frac{T}{2} - \frac{1}{1+n}\sin\frac{2\pi}{T}(1+n)\frac{T}{2}\right\}$$

$$= \frac{A}{2}\left\{\frac{1}{1-n}\sin\left[\pi(1-n)\right] - \frac{1}{1+n}\sin\left[\pi(1+n)\right]\right\}$$

$b_n = 0$ for all n except $n=1$

$n=1$, $b_1 = \frac{2A}{T}\int_0$

$n=1$, $b_1 = \frac{2A}{T}\int_0^{T/2} \sin^2\left(\frac{2\pi t}{T}\right) dt$

$= \frac{2A}{T}\int_0^{T/2} \frac{1}{2}\left[1-\cos\left(\frac{4\pi t}{T}\right)\right] dt$

$= \frac{2A}{T}\left[t - \sin\left(\frac{4\pi t}{T}\right)\times\frac{T}{4\pi}\right]_0^{T/2}$

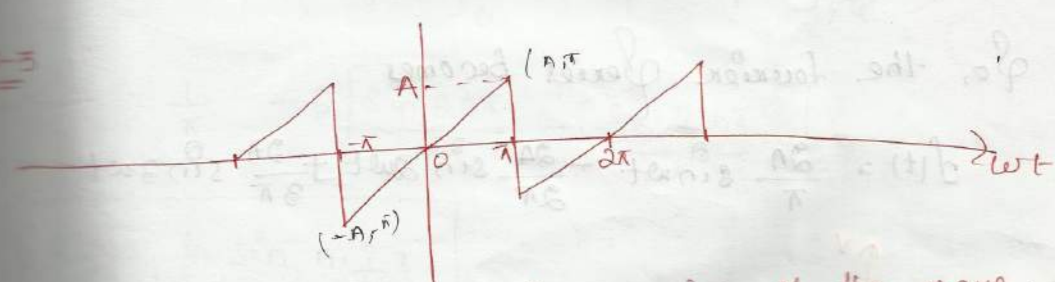
$= \frac{2A}{T}\left(\frac{T}{2} - \frac{T}{4\pi}\sin\frac{4\pi}{T}\times\frac{T}{2}\right)$

$= \frac{A}{2}\left[1 - \frac{1}{2\pi}\sin 2\pi\right]$

$\Rightarrow \boxed{b_1 = \frac{A}{2}}$

So, $f(t) = \frac{A}{\pi} + \frac{A}{2}\sin\left(\frac{2\pi t}{T}\right) - \frac{2A}{3\pi}\cos\left(\frac{4\pi t}{T}\right) - \frac{2A}{15\pi}\cos\left(\frac{8\pi t}{T}\right) - - - \cdot$

Ex-3



Find Determine fourier series of the wave.

Sol:

As the given function is odd symmetry.

So, $a_0 = a_n = 0$

$f(t) = \frac{A}{\pi}\omega t$

$$b_n = \frac{2}{2\pi}\int_{-\pi}^{\pi} \frac{A}{\pi}\,\omega t\,\sin n\omega t \; d(\omega t)$$

$$= \frac{A}{\pi^2}\left[-\omega t\,\frac{\cos n\omega t}{n} + \int \frac{\cos n\omega t}{n}\,dt\right]$$

$$= \frac{A}{\pi^2}\left[\frac{-\omega t}{n}\cos n\omega t + \frac{1}{n^2}\sin n\omega t\right]_{-\pi}^{\pi}$$

$$= \frac{A}{\pi^2}\left[-\frac{\pi}{n}\cos n\pi + \frac{1}{n^2}\sin n\pi - \frac{\pi}{n}\cos n\pi + \frac{1}{n^2}\sin n\pi\right]$$

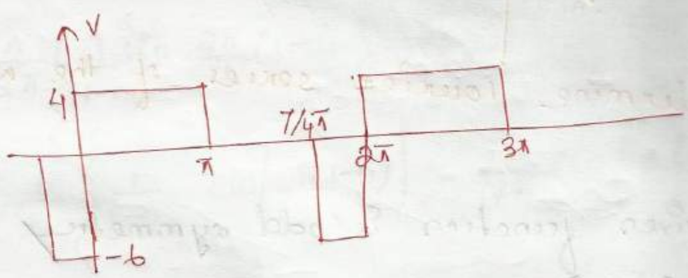
$$= -\frac{2A}{\pi^2} \times \frac{\pi}{n}\cos n\pi$$

$$\Rightarrow \boxed{b_n = -\frac{2A}{n\pi}\cos n\pi}$$

So, the Fourier series becomes

$$f(t) = \frac{2A}{\pi}\sin\omega t - \frac{2A}{2\pi}\sin 2\omega t + \frac{2A}{3\pi}\sin 3\omega t - - -$$

Ex-4



Find a & b co-efficients of Fourier series.

Sol^n

$$a_0 = \frac{1}{T}\int f(t)\,dt$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} 4\, d(\omega t) + \int_{7/4\pi}^{2\pi} -6\, d(\omega t)\right]$$

$$= \frac{1}{2\pi}\left\{4\left[\omega t\right]_0^{\pi} - 6\left[\omega t\right]_{7/4\pi}^{2\pi}\right\}$$

$$= \frac{1}{2\pi}\times 4\pi - \frac{6}{2\pi}\left(2\pi - \frac{7}{4}\pi\right)$$

$$= 2 - \frac{3}{\pi}\times\frac{\pi}{4} = 2 - \frac{3}{4} = \frac{5}{4} = 1.25$$

$$a_n = \frac{2}{2\pi}\left[\int_0^{\pi} 4\cos n\omega t\, d\omega t + \int_{7/4\pi}^{2\pi} -6\cos n\omega t\, d(\omega t)\right]$$

$$= \frac{1}{\pi}\left\{4\left[\frac{\sin n\omega t}{n}\right]_0^{\pi} - 6\left[\frac{\sin n\omega t}{n}\right]_{7/4\pi}^{2\pi}\right\}$$

$$= \frac{1}{\pi}\; -6\left[\frac{\sin 2n\pi}{n} - \frac{\sin \frac{7}{4}n\pi}{n}\right]$$

$$\boxed{a_n = \frac{6}{n\pi}\sin n\cdot\frac{7}{4}\pi}$$

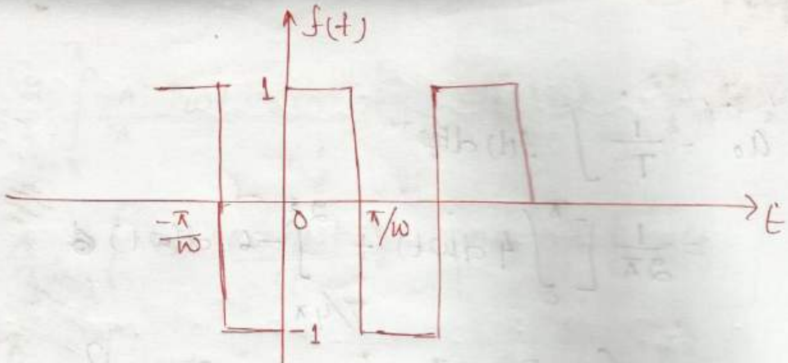
$$b_n = \frac{2}{2\pi}\left\{\int_0^{\pi} 4\sin n\omega t\, d\omega t - \int_{7/4\pi}^{2\pi} 6\sin n\omega t\, d\omega t\right\}$$

$$= \frac{1}{\pi}\left\{4\left[-\frac{\cos n\omega t}{n}\right]_0^{\pi} + 6\left[\frac{\cos n\omega t}{n}\right]_{7/4\pi}^{2\pi}\right\}$$

$$= \frac{1}{\pi}\left[\frac{-4}{n}\left(\cos n\pi - 1\right) + \frac{6}{n}\left(\cos 2n\pi - \cos\frac{7}{4}n\pi\right)\right]$$

$$= \frac{1}{n\pi}\left[-4\left(\cos n\pi - 1\right) + 6\left(1 - \cos\frac{7}{4}n\pi\right)\right]$$

Ex-5



Find the complex Fourier co-efficient of the waveform.

Sol^n

$$f(t) = -1 \quad \text{for } -\frac{\pi}{\omega} < t < 0$$
$$= 1 \quad \text{for } 0 < t < \pi/\omega$$

$$C_n = \frac{1}{T_0}\int f(t)\, e^{-jn\omega t}\,dt$$

$$= \frac{\omega}{2\pi}\left[\int_{-\pi/\omega}^{0}(-1)\,e^{-jn\omega t}\,dt + \int_{0}^{\pi/\omega}(1)\,e^{-jn\omega t}\,dt\right]$$

$$= \frac{\omega}{2\pi}\left\{\left[\frac{-e^{-jn\omega t}}{-jn\omega}\right]_{-\pi/\omega}^{0} + \left[\frac{e^{-jn\omega t}}{-jn\omega}\right]_{0}^{\pi/\omega}\right\}$$

$$= \frac{1}{2jn\pi}\left\{\left[1 - e^{jn\omega \times \frac{\pi}{\omega}}\right] - \left(e^{-jn\omega\frac{\pi}{\omega}} - 1\right)\right\}$$

~~$= \frac{1}{2jn\pi}\left[2\left(1 - e^{-jn\pi}\right)\right]$~~

$$= \frac{1}{2jn\pi}\left[2 - \left(e^{jn\pi} + e^{-jn\pi}\right)\right]$$

$$\boxed{C_n = \frac{1 - \cos n\pi}{jn\pi}}$$

$C_n = 0$, for n even

$= \frac{2}{jn\pi}$ for n odd.

RMS value of Periodic Complex wave (Non-sinusoidal)

The rms value or x_{rms} for any periodic function is

$$x_{rms} = \sqrt{\frac{1}{T}\int_0^T x^2(t)\,dt}$$

Generally the periodic non-sinusoidal complex voltage wave is represented by

$$e = E_0 + E_{max1}\sin(\alpha+\phi_1) + E_{max2}\sin(2\alpha+\phi_2) + \cdots + E_{maxn}\sin(n\alpha+\phi_n)$$

where α is any variable.

So,

$$E_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}\left[E_0 + E_{max1}\sin(\alpha+\phi_1) + \cdots + E_{maxn}\sin(n\alpha+\phi_n)\right]^2}$$

From the power value calculation,
we know that

$$\frac{1}{2\pi}\int_0^{2\pi} E_{maxn}^2 \sin^2(n\alpha+\phi_n)\,d\alpha = \frac{E_{maxn}^2}{2} \quad n=1,2\cdots$$

So,

$$E_{rms} = \sqrt{E_0^2 + \frac{E_{max1}^2}{2} + \frac{E_{max2}^2}{2} + \cdots + \frac{E_{maxn}^2}{2}}$$

$$= \sqrt{E_0^2 + \left(\frac{E_{max1}}{\sqrt{2}}\right)^2 + \left(\frac{E_{max2}}{\sqrt{2}}\right)^2 + \cdots + \left(\frac{E_{maxn}}{\sqrt{2}}\right)^2}$$

$$= \sqrt{E_0^2 + E_1^2 + E_2^2 + \cdots + E_n^2}$$

Where $E_1, E_2 \cdots E_n$ are the rms value of the harmonic components of the wave.

Expression of Power with Non-Sinusoidal voltage & current

The non-sinusoidal voltage-wave is expressed as

$$e = E_0 + E_{max1}\sin(\alpha+\phi_1) + E_{max2}\sin(2\alpha+\phi_2) + \cdots + E_{maxn}\sin(n\alpha+\phi_n) \quad \text{--- (1)}$$

Similarly, the non-sinusoidal current wave is expressed as

$$i = I_0 + I_{max1}\sin(\alpha+\phi_1+\psi_1) + I_{max2}\sin(2\alpha+\phi_2+\psi_2) + \cdots + I_{maxn}\sin(n\alpha+\phi_n+\psi_n) \quad \text{--- (2)}$$

The average power P is given by

$$P = \frac{1}{2\pi}\int_0^{2\pi} e\cdot i \, d\alpha \quad \text{--- (3)}$$

However eq^n (1) & (2) can be written as

$$e = E_0 + \sum_{n=1}^{n} E_{maxn}\sin(n\alpha+\phi_n) \quad \text{--- (4)}$$

$$i = I_0 + \sum_{n=1}^{n} I_{maxn}\sin(n\alpha+\phi_n+\psi_n) \quad \text{--- (5)}$$

So, eq^n (3) become

$$P = \frac{1}{2\pi}\int_0^{2\pi}\left[\left\{E_0 + \sum_{n=1}^{n} E_{maxn}\sin(n\alpha+\phi_n)\right\} \times \left\{I_0 + \sum_{n=1}^{n} I_{maxn}\sin(n\alpha+\phi_n+\psi_n)\right\}\right]d\alpha$$

$$= \frac{1}{2\pi}\int_0^{2\pi}\left[E_0 I_0 + E_0\sum_{n=1}^{n} I_{maxn}\sin(n\alpha+\phi_n+\psi_n) + I_0\sum_{n=1}^{n} E_{maxn}\sin(n\alpha+\phi_n) + \sum_{n=1}^{n} E_{maxn}\sin(n\alpha+\phi_n) \times \sum_{n=1}^{n} I_{maxn}\sin(n\alpha+\phi_n+\psi_n)\right]d\alpha$$

$$\Rightarrow P = \frac{1}{2\pi}\left[E_0 I_0 \times 2\pi + \frac{1}{2}\int_0^{2\pi} \cos\psi_n \, d\alpha\right]$$

$$\Rightarrow P = \frac{1}{2\pi}\left[E_0 I_0 \times 2\pi + \frac{1}{2}\int_0^{2\pi} E_{max\,n} I_{max\,n} \cos\psi_n \, d\alpha\right]$$

$$= \frac{1}{2\pi}\left[E_0 I_0 \times 2\pi + \frac{E_{max\,n} I_{max\,n}}{2} \cos\psi_n \times 2\pi\right]$$

$$\Rightarrow P = E_0 I_0 + \frac{E_{max\,n} I_{max\,n}}{2} \cos\psi_n$$

$$\Rightarrow P = E_0 I_0 + \frac{E_{max\,1} I_{max\,1}}{2} \cos\psi_1 + \frac{E_{max\,2} I_{max\,2}}{2} \cos\psi_2 + \cdots$$

Properties of Network Functions

A network function exhibits the relationship between the transform of the source or excitation to the transform of the response for a electrical network.

Driving point Impedance & Admittance

The driving point impedance of a one port network is defined as

$$Z(s) = \frac{V(s)}{I(s)}$$

While driving point admittance is

$$Y(s) = \frac{I(s)}{V(s)}$$

For Two port N/W, the driving point impedance & admittance at port 1 is defined as

$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)}$$

$$Y_{11}(s) = \frac{I_1(s)}{V_1(s)}$$

While driving point impedance & admittance at port 2 is defined as

$$Z_{22}(s) = \frac{V_2(s)}{I_2(s)}$$

$$Y_{22}(s) = \frac{I_2(s)}{V_2(s)}$$

Transfer Impedance & Admittance

Transfer Impedance is defined as the ratio of transformed voltage at output to the transformed current at the input port of a two port N/W.

i.e $$Z_{12}(s) = \frac{V_2(s)}{I_1(s)}$$

Similarly, transfer admittance is defined as the ratio of current transform at output port to the voltage transform at the input port.

i.e $$Y_{12}(s) = \frac{I_2(s)}{V_1(s)}$$

Voltage & current Transfer Ratio

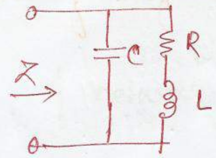
Voltage transfer ratio is the ratio of voltage transform at output port to the voltage transform at the input port.

i.e $$G_{12}(s) = \frac{V_2(s)}{V_1(s)}$$

Similarly Current transfer ratio is

$$\alpha_{12}(s) = \frac{I_2(s)}{I_1(s)}$$

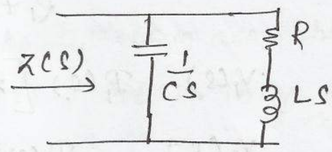
Ex-1



Find the i/p impedance in Laplace domain.

Soln

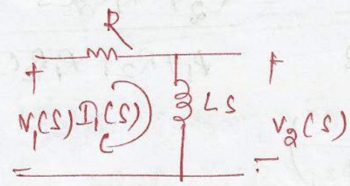
$$Z(s) = \frac{\frac{1}{Cs}(R+Ls)}{\frac{1}{Cs}+R+Ls}$$



$$= \frac{R+Ls}{1+RCs+LCs^2}$$

$$Z(s) = \frac{(s+\frac{R}{L})}{C\left(s^2+\frac{R}{L}s+\frac{1}{LC}\right)}$$

Ex-2



Find $Z_{11}(s)$ & $Z_{12}(s)$.

Soln

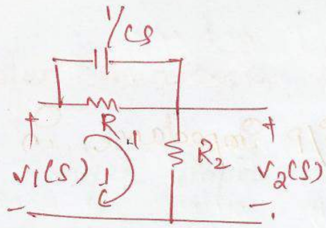
$$Z_{11}(s) = R+Ls$$

$$V_2(s) = I_1(s)\,Ls$$

$$Z_{12}(s) = \frac{V_2(s)}{I_1(s)} = Ls$$

Ex

Ex-3



Obtain transfer function $\frac{V_2(S)}{V_1(S)}$

Sol^n

$$Z(s) = \frac{R_1 \times \frac{1}{Cs}}{R_1 + \frac{1}{Cs}}$$

$$V_1(s) = I_1(s)\left[Z(s) + R_2\right]$$

$$V_2(s) = I_1(s) R_2$$

$$G_{12}(s) = \frac{V_2(s)}{V_1(s)} = \frac{R_2}{\frac{R_1 \times \frac{1}{Cs}}{R_1 + \frac{1}{Cs}} + R_2} = \frac{R_2}{\frac{R_1}{R_1 Cs + 1} + R_2}$$

$$= \frac{R_2(R_1 Cs + 1)}{R_1 + R_2 R_1 Cs + R_2} = \frac{R_2 + R_1 R_2 Cs}{R_1 + R_2 + R_1 R_2 Cs}$$

$$= \frac{R_1 R_2 C\left(s + \frac{1}{R_1 C}\right)}{R_1 R_2 C\left(s + \frac{1}{R_2 C} + \frac{1}{R_1 C}\right)}$$

$$\Rightarrow \boxed{G_{12}(s) = \frac{\left(s + \frac{1}{R_1 C}\right)}{\left(s + \frac{1}{R_1 C} + \frac{1}{R_2 C}\right)}}$$

Concept of Poles and zeros in a network function

A network function $H(s)$ may be written as

$$H(s) = \frac{A(s)}{B(s)} = \frac{a_0 s^n + a_1 s^{n-1} + a_2 - - \cdot + a_{n-1} s + a_n}{b_0 s^m + b_1 s^{m-1} + - - - + b_{m-1} s + b_m}$$

where $a_0, a_1, a_2 \cdots a_n$ and $b_0, b_1, \cdots, b_m$ are the coefficients of the polynomials $A(s)$ & $B(s)$. They are real and +ve for a passive network.

Factorising the numerator and denominator, the network function can be written as

$$H(s) = \frac{A(s)}{B(s)} = \frac{a_0 (s - z_1)(s - z_2)(s - z_3) \cdots (s - z_n)}{b_0 (s - P_1)(s - P_2)(s - P_3) \cdots (s - P_n)}.$$

Where $z_1, z_2 \cdots, z_n$ are the nth roots for $A(s) = 0$

$P_1, P_2 \cdots P_m$ are the n roots for $B(s) = 0$.

$K = \left(\frac{a_0}{b_0}\right)$ is a constant known as scale factor.

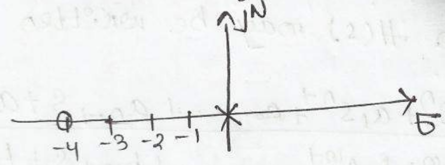
Here $z_1, z_2 \cdots z_n$ are called "zeros" and denoted by a "small circle" while $P_1, P_2 \cdots P_m$ are called "Poles" and denoted by a "cross".

The network function $H(s)$ become zero if "s" is equal to any of the zeros and become infinity when s is equal to any of the poles.

Ex- A function $Z(s) = \frac{s+4}{s}$. Find the pole-zero plot.

Sol^n

pole = 0
zero = −4



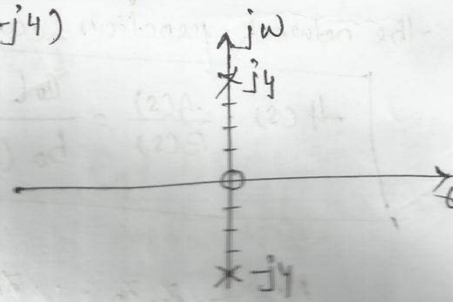
Ex $Z(s) = \frac{2s}{s^2+16}$. Draw its pole-zero plot.

Sol^n

$s^2+16 = (s+j4)(s-j4)$

So, $Z(s) = \frac{2s}{(s+j4)(s-j4)}$

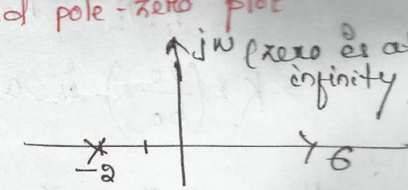
Zero = 0
Poles = $+j4$ & $-j4$



Ex $Y(s) = \frac{1}{s+2}$. Find pole-zero plot.

Sol^n

Pole = −2
Zero is at infinity



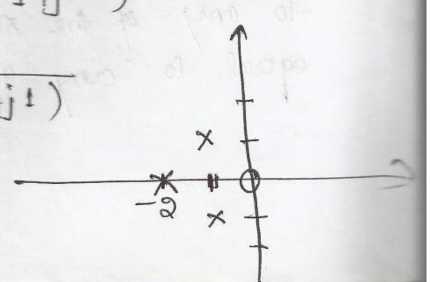
Ex $Z(s) = \frac{2s}{(s+2)(s^2+2s+2)}$. Find pole-zero plot.

Sol^n

$s^2+2s+2 = (s+1-j1)(s+1+j1)$

$Z(s) = \frac{2s}{(s+2)(s+1-j1)(s+1+j1)}$

Zero = 0
Poles = $-2, -(1-j1), -(1+j1)$



Restriction on Location of poles & zeros in driving point function

(1) The co-efficients of the polynomials $A(s)$ & $B(s)$ of the network function $H(s)$ must be real & positive.

(2) Poles & zeros, if complex or imaginary, must occur in conjugate pairs.

(3) The real parts of all poles and zero must be zero or negative.

(4) The polynomial $A(s)$ or $B(s)$ cannot have any missing term between those of highest and lowest order values unless all even order or all odd order terms are missing.

(5) The degree of $A(s)$ and $B(s)$ may differ by zero or one only.

(6) The lowest degree in $A(s)$ and $B(s)$ may differ in degree by at the most one.

Ex $Z(s) = \dfrac{s^4+s+1}{s^3+2s^2-2s+10}$ can represent a passive one port N/W

Sol

The given function is not suitable to represent the impedance of a one port N/W.

Bcz (i) In the numerator, one coefficient is missing

(ii) In the denominator, one co-efficient is negative.

Synthesis of Passive Network

Necessary Conditions of Stability of Network Function [F(s)]

For a network function to be stable, the following three conditions must be satisfied -

(i) F(s) can't have poles in the right-half of s-plane.

(ii) F(s) should not have any multiple poles in the $j\omega$ axis.

(iii) The degree of the numerator of F(s) can't exceed the degree of the denominator by more than unity.

i.e $F(s) = \dfrac{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}$

then the order of n can't exceed the order of m by more than unity i.e $n - m \leq 1$.

However, if $n - m > 1$, it would simple mean multiple poles at $s = \infty$, which impairs the stability.

Hurwitz Polynomials

The denominator polynomial P(s) or B(s) of the system function is termed as Hurwitz polynomial.

Then a polynomial is said to be Hurwitz iff

(i) $P(s)$ is real when s is real.

(ii) The roots of $P(s)$ have real parts which are to be zero or negative.

Properties

1) Between the highest order term in s and the lowest order term, none of the co-efficients may be zero (unless the polynomical is even or odd.

We know that

$$P(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0$$

Separating the even & odd parts of $P(s)$

$$M(s) = a_n s^n + a_{n-2} s^{n-2} + \cdots$$

$$N(s) = a_{n-1} s^{n-1} + a_{n-3} s^{n-3} + \cdots$$

where, for n even, $M(s)$ is even and $N(s)$ odd and for n odd, $M(s)$ is odd and $N(s)$ is even,

Next, we obtain the continued fraction of $P(s) = \frac{M(s)}{N(s)}$ by successive division & inversion as follows -

$$\frac{M(s)}{N(s)} = \frac{a_n s^n + a_{n-2} s^{n-2} + \cdots}{a_{n-1} s^{n-1} + a_{n-3} s^{n-3} + \cdots}$$

$$= \frac{a_n}{a_{n-1}} s + \frac{a'_{n-2} s^{n-2} + a'_{n-4} s^{n-4} + \cdots}{a_{n-1} s^{n-1} + a_{n-3} s^{n-3} + \cdots}$$

$$= \alpha_1 s + \frac{M_1(s)}{N(s)}$$

$\alpha_1 s$ is the quotient resulting from 1st division & $M_1(s)/N(s)$ is the remainder.

Inverting the remainder & divide again

$$\frac{M(s)}{N(s)} = \alpha_1 s + \frac{1}{N(s)/M_1(s)} = \alpha_1 s + \frac{1}{\alpha_2 s + \frac{N_2(s)}{M_1(s)}}$$

The process of division & inversion is repeated until the function M(s)/N(s) is exhausted. The result is continued fraction expansion of M(s)/N(s) of the form

$$\frac{M(s)}{N(s)} = \alpha_1 s + \frac{1}{\alpha_2 s + \frac{1}{\alpha_3 s + \ddots \frac{}{\alpha_n s}}}$$

Procedure of Testing of a given polynomial for Hurwitz ~~chacter~~ character —

Physical testing

(1) All the co-efficients of the polynomial must be positive and real.

(2) There must not be any power of s missing between the highest degree & lowest degree of the polynomial (unless the polynomial is completely even or completely odd)

Analytic Testing

(a) The quotients $(\alpha_1, \alpha_2 \cdots)$ in the continued fraction expansion of $\chi(s)$ $\left[= \frac{M(s)}{N(s)}\right]$ must be real & positive. [M(s) must be even & N(s) odd]

If due to common factor bet^n M(s) & N(s), if the continued fraction is prematurely terminated, then the quotients in the continued fraction expansion of $\psi(s)$ $\left[\psi(s) = \frac{P(s)}{P'(s)}\right.$, $P'(s)$ being 1st derivative of $P(s)$] must be real & positive.

Use of $\psi(s)$ is also suitable if the given polynomial is either only even or only odd.

Ex-1 Check whether the polynomial $s^5+9s^4+7s^3+s^2+4s$ is Hurwitz or not.

Sol^n

$$P(s) = s^5 + 9s^4 + 7s^3 + s^2 + 4s$$

$$N(s) = s^5 + 7s^3 + 4s = s(4 + 7s^2 + s^4)$$

$$M(s) = 9s^4 + s^2 = s^2(1 + 9s^2)$$

$$\chi(s) = \frac{N(s)}{M(s)} = \frac{4 + 7s^2 + s^4}{s(1+9s^2)}$$

Ex-2 $P(s) = s^4 + s^3 + 2s^2 + 3s + 2$. check the polynomial Hurwitz or not.

Sol^n

$M(s) = s^4 + 2s^2 + 2$

$N(s) = s^3 + 3s$

$s^3+3s \,\big)\, s^4 + 2s^2 + 2 \,\big(\, s$
$\quad s^4 + 3s^2$
$\quad \overline{-s^2 + 2}$

$-s^2+2 \,\big)\, s^3 + 3s \,\big(\, -s$
$\quad s^3 - 2s$
$\quad \overline{5s}$

$5s \,\big)\, -s^2 + 2 \,\big(\, -\frac{s}{5}$
$\quad -s^2$
$\quad \overline{2}$

$2 \,\big)\, 5s \,\big(\, \frac{5s}{2}$
$\quad 5s$
$\quad \overline{\times}$

Since the continued fraction contain negative quotients hence the given polynomial is not Hurwitz.

Ex $P(s) = s^4 + 11s^3 + 39s^2 + 51s + 20$.

$M(s) = s^4 + 39s^2 + 20$

$N(s) = 11s^3 + 51s$

$$11s^3+51s \,\big|\, s^4+39s^2+20 \,\big|\, \frac{s}{11}$$
$$s^4+\frac{51}{11}s^2$$

$$\frac{378}{11}s^2+20 \,\big|\, 11s^3+51s \,\big|\, \frac{121}{378}s$$
$$11s^3+\frac{1210}{189}s$$

$$\frac{8429}{189}s \,\big|\, \frac{378}{11}s^2+20 \,\big|\, \frac{378\times 189}{8429\times 11}s$$
$$\frac{378}{11}s^2$$

$$20 \,\big|\, \frac{8429}{189}s \,\big|\, s\times\frac{8429}{189\times 20}$$
$$\frac{8429}{189}s$$

$$\times$$

Since the continued fraction has all +ve quotients hence the given polynomial is Hurwitz.

Q

a(1) $P(s) = s^4+7s^3+4s^2+18s+6$ (not)

(2) $P(s) = s^4+s^3+2s^2+4s+1$ (not)

(3) $P(s) = s^4+s^3+6s^2+3s+6$ (yes)

Ex-4 $P(s) = s^5+s^3+s$, Check whether Hurwitz or not.

Sol^n

$P(s) = s^5+s^3+s$, consists of only odd function

So, $P'(s) = 5s^4+3s^2+1$

Now $\psi = \frac{P(s)}{P'(s)}$

$$5s^4+3s^2+1 \,\Big|\, s^5+s^3+s \,\Big|\, \frac{s}{5}$$
$$s^5+\frac{3}{5}s^3+\frac{s}{5}$$
$$\overline{\frac{2}{5}s^3+\frac{4}{5}s} \,\Big|\, 5s^4+3s^2+1 \,\Big|\, \frac{25}{2}s$$
$$5s^4+10s^2$$
$$\overline{-7s^2+1} \,\Big|\, \frac{2}{5}s^3+\frac{4}{5}s \,\Big|\, -\frac{2}{35}s$$
$$\frac{2}{5}s^3-\frac{2}{35}s$$
$$\overline{\frac{6}{7}s} \,\Big|\, -7s^2+1 \,\Big|\, -\frac{49}{6}s$$
$$-7s^2$$
$$\overline{1} \,\Big|\, \frac{6}{7}s \,\Big|\, \frac{6s}{7}$$
$$\frac{6s}{7}$$
$$\overline{\times}$$

∴ Polynomial is not Hurwitz.

Q (1) $P(s) = s^4+3s^2+2$ (yes)

(2) $P(s) = s^5+3s^4+3s^3+4s^2+s+1$ (yes)

Ex-5 $P(s) = s^8+3s^7+10s^6+24s^5+35s^4+57s^3+50s^2+36s+24$

Solⁿ

$M(s) = s^8+10s^6+35s^4+50s^2+24$

$N(s) = 3s^7+24s^5+57s^3+36s$

$$3s^7+24s^5+57s^3+36s \,\Big|\, s^8+10s^6+35s^4+50s^2+24 \,\Big|\, \frac{s}{3}$$
$$s^8+8s^6+19s^4+12s^2$$
$$\overline{2s^6+16s^4+38s^2+24} \,\Big|\, 3s^7+24s^5+57s^3+36s \,\Big|\, \frac{3}{2}s$$
$$3s^7+24s^5+57s^3+36s$$
$$\overline{\times}$$

Though the 1st two quotients are +ve, but the process has terminated after 2 steps. Thus it is evident that M(s) & N(s) have the even polynomial $(2s^6+16s^4+38s^2+24)$ as a common factor.

So, $E(s) = s^6+8s^4+19s^2+12$

$E'(s) = 6s^5+32s^3+38s$

$$6s^5+32s^3+38s \,\Big|\, s^6+8s^4+19s^2+12 \,\Big|\, \frac{s}{6}$$
$$s^6+\frac{16}{3}s^4+\frac{19}{2}s^2$$
$$\frac{8}{3}s^4+\frac{19}{2}s^2+2 \,\Big|\, 6s^5+3s^2+38s \,\Big|\, \frac{9}{4}s$$
$$6s^5+$$

$\frac{19}{38}$ $\frac{28}{3}\times\frac{9}{4}$

Positive Real (PR) Functions

The driving point impedance function [Z(s)] as well as driving point admittance function [Y(s)] of one port network can be represented in the form of

$$F(s) = \frac{A(s)}{B(s)} = \frac{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n}{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1}s + b_m}$$

The function F(s) is called a positive real (or PR) function iff

(a) F(s) is real for s real.

(b) B(s) is Hurwitz polynomial.

(c) If F(s) has poles on (jw) axis, the poles are simple and the residues thereof are real & positive.

(d) Real $F(j\omega) \geq 0$ for all value of ω.

~~[illegible]~~

Properties

Let $F(s) = A(s)/B(s)$

(i) Both A(s) & B(s) polynomials are Hurwitz, and the poles & zeros of a PR function can't have +ve real parts.

(ii) The highest and lowest powers of A(s) & B(s) differ by one unity.

(iii) If F(s) is a PR function, the reciprocal is also PR function.

(iv) The sum of PR function is also PR function, but the difference may not be.

Requirements of a PR function

(1) Function being of type $F(s) = \frac{s+\alpha}{s^2+\beta s+\gamma}$, α, β & γ being real, F(s) will be PR function iff $\alpha, \beta, \gamma \geq 0$ and $\beta \geq \alpha$.

(2) Function being of type $F(s) = \frac{Ks}{s^2+a}$, a and K being real, F(s) will be a PR function iff $a, K \geq 0$.

(3) Function being a type of $F(s) = \frac{s+b}{s+a}$, a, b being real, F(s) will be a PR function iff $a, b \geq 0$.

(4) Function being of type $F(s) = \frac{s^2+a_1 s+a_0}{s^2+b_1 s+b_0}$, a_0, a_1, b_0 & b_1 being real, F(s) will be a PR function iff a_0, a_1, b_0 & $b_1 \geq 0$ & $a_1 b_1 \geq \left(\sqrt{a_0} - \sqrt{b_0}\right)^2$

Necessary but not sufficient conditions of PR function

(1) All the coefficients of the polynomial must be real & positive.

(2) Imaginary axis poles & zero must be simple.

(3) Degree of numerator & denominator polynomial may differ by at most unity.

(4) Terms in the lowest degree of numerator and denominator polynomial may differ by at most unity.

(5) Unless the polynomial is either even or odd completely, there would be no missing term between the highest & lowest degree is numerator & denominator polynomial.

Necessary & sufficient conditions of PR function

(1) $F(s)$ must have simple poles on $j\omega$ axis with real & positive residues.

(2) If $F(s) = \frac{P(s)}{Q(s)}$, then $P(s) + Q(s)$ must be Hurwitz

(3) If $F(s) = \frac{P(s)}{Q(s)} = \frac{M_1(s) + N_1(s)}{M_2(s) + N_2(s)}$, M being the even parts & N being the odd parts, then for the positive realness of $F(s)$, $M_1 M_2 - N_1 N_2 \big|_{s=j\omega} \geq 0$ for all ω such that $Re[F(j\omega)] \geq 0$ for all ω.

Ex Check whether $F(s) = \frac{s+2}{s+1}$ is a PR function.

Sol^n (i) Since all the quotient terms of $F(s)$ are real hence $F(s)$ is real if s is real.

(ii) Poles & zero of the function lie on the left half of the s-plane.

(iii) $$Re[F(j\omega)] = Re\left[\frac{j\omega + 2}{j\omega + 1}\right] \times \left[\frac{-j\omega + 1}{-j\omega + 1}\right]$$

$$= Re\left[\frac{\omega^2 + j\omega - 2j\omega + 2}{\omega^2 + 1}\right] = \frac{\omega^2 + 2}{\omega^2 + 1}$$

So, for all values of ω, $Re[F(j\omega)] \geq 0$

Hence the given function is a PR function.

Ex check the Positive realness of the function.

$$F(s) = \frac{s^2+10s+4}{s+2}$$

Sol^n

(a) All the coefficients are positive.

(b) Poles & zero present in the left half of the s-plane.

(c) $Re[F(j\omega)] = Re\left[\frac{-\omega^2+10j\omega+4}{j\omega+2}\right]\left[\frac{-j\omega+2}{-j\omega+2}\right]$

$$= Re\left[\frac{-2\omega^2+20j\omega+8+j\omega^3+10\omega^2-4j\omega}{\omega^2+4}\right]$$

$$= Re\left[\frac{8\omega^2+16j\omega+j\omega^3+8}{\omega^2+4}\right]$$

$$= Re\left[\frac{(8\omega^2+8)+j(\omega^3+16\omega)}{\omega^2+4}\right]$$

$$= \frac{8\omega^2+8}{\omega^2+4}$$

Since for all values of ω, $Re[F(j\omega)] \geq 0$.

So, the function is a PR function.

Q Check the positive realness of the function

$$Y(s) = \frac{s^2+2s+20}{s+10}$$

Ex $Z(s) = \dfrac{s^3+5s^2+9s+3}{s^3+4s^2+7s+9}$. Find the positive realness of the function.

Sol^n

(i) All the coefficients are +ve, so $Z(s)$ is real.

(ii) $B(s)$ should be Hurwitz polynomial.

so, ~~Z(s) M(s)~~ $B(s) = s^3+4s^2+7s+9$

$M(s) = 4s^2+9$

$N(s) = s^3+7s$

$$
\begin{array}{r|l|l}
4s^2+9 & s^3+7s & \frac{s}{4} \\
 & s^3+\frac{9}{4}s & \\
\hline
\frac{19}{4}s & 4s^2+9 & \frac{16}{19}s \\
 & 4s^2 & \\
\hline
9 & \frac{19}{4}s & \frac{19}{36}s \\
 & \frac{19}{4}s & \\
\hline
 & \times &
\end{array}
$$

So, $B(s)$ is Hurwitz polynomial.

(iii) $Re[Z(j\omega)] \geq 0$.

Let $Z(s) = \dfrac{M_1(s)+N_1(s)}{M_2(s)+N_2(s)}$

$M_1(s) = 5s^2+3 \qquad N_1(s) = s^3+9s$

$M_2(s) = 4s^2+9 \qquad N_2(s) = ~~9s^2~~ +7s$

~~Realis~~

Rationalising $Z(s) = \dfrac{M_1+N_1}{M_2+N_2} \cdot \dfrac{M_2-N_2}{M_2-N_2}$

$$= \frac{M_1M_2 - M_1N_2 + M_2N_1 - N_1N_2}{M_2^2 - N_2^2}$$

$$= \frac{M_1M_2 - N_1N_2}{M_2^2 - N_2^2} + \frac{N_1M_2 - M_1N_2}{M_2^2 - N_2^2}$$

even part of $Z(s) = \dfrac{M_1M_2 - N_1N_2}{M_2^2 - N_2^2}$

Let $D(s) = M_1M_2 - N_1N_2$

$= (5s^2+3)(4s^2+9) - (s^3+9s)(s^3+7)$

$= 20s^4 + 45s^2 + 12s + 27 - s^6 - 7s^4 - 9s^4 - 63s^2$

$= -s^6 + 4s^4 - 6s^2 + 27$

$D(j\omega) = -(j\omega)^6 + 4(j\omega)^4 - 6(j\omega)^2 + 27$

$= \omega^6 + 4\omega^4 + 6\omega^2 + 27$.

i.e $D(j\omega) > 0$ for any value of ω.

Thus $Z(j\omega) \geq 0$ for any value of ω.

$\therefore$ Given function is PR funct^n.

Ex check whether the function

$$Z(s) = \frac{2s^2+2s+1}{s^3+2s^2+s+2}$$ is a PR funct^n or not

sol^n

(i) All the coefficients are +ve.

(ii) $s^3+2s^2+s+2 = s^2(s+2)+1(s+2)$

$= (s^2+1)(s+2)$

So, the poles of the function are $\pm j1$ & -2

Since the poles are on $j\omega$ axis, we have to determine the residue.

Let $Z(s) = \frac{A \cdot s}{s^2+1} + \frac{B}{s+2}$

$$A = \left.\frac{2s^2+2s+1}{s(s+2)}\right|_{s^2=-1} = \frac{-2+2s+1}{-2+2s} = \left.\frac{2s-1}{2s-2}\right|_{s^2=-1}$$

$$= \frac{(2s-1)(2s+2)}{4s^2-4} = \frac{4s^2+2s-2s-2}{-4-4} = \frac{-4-2}{-8}$$

$$B = \left.\frac{2s^2+2s+1}{s^2+1}\right|_{s=-2} = \frac{8-4+1}{4+1} = \frac{5}{5} = 1$$

The residue is +ve.

(iii) $D(s) = M_1M_2 - N_1N_2 = (2s^2+1)(2s^2+2) - 2s(s^2+1)$

$= 2(s^2+1)^2$

$D(j\omega) = 2(-\omega^2+1)^2$

The $D(j\omega) \geq 0$ for all value of ω.

Network Synthesis

Procedure for Synthesis

Ex-1 $Z(s) = \frac{s^2+4s}{s^2+2}$, Realise the network.

Sol^n

Step-1:- Since the degree of numerator polynomial is one higher than that of denominator, hence it is evident that $Z(s)$ will have a pole at $s=\infty$ indicating the presence of a series inductor whose value can be determined by long-division of the numerator of $Z(s)$ by its denominator.

Step-2

$$s^2+2 \,\big)\, s^3+4s \,\big(\, s$$
$$\underline{s^3+2s}$$
$$2s$$

$$Z(s) = s + \frac{2s}{s^2+2} = Z_1(s) + Z_2(s)$$

Thus $Z_1(s) = [1 \cdot s]$, indicates that the series inductance would have value of 1H.

Step-3

Since $Z_2(s) = \frac{2s}{s^2+2}$, $Y_2(s) = \frac{s^2+2}{2s}$

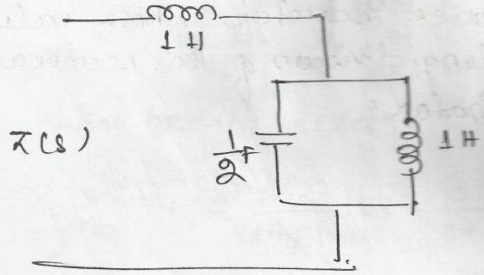
Presence of pole at $s=\infty$ is evident as the degree of numerator is still one high. for the admitance function $Y_2(s)$, presence of pole at $s=\infty$ indicates a parallel capacitance.

Step-4

$$2s \,\Big|\, s^2+2 \,\Big|\, \frac{s}{2}$$
$$\underline{s^2}$$
$$2$$

So, $Y_2(s) = \frac{s}{2} + \frac{1}{s} = Y_3(s) + Y_4(s)$

Here $Y_3(s) = \frac{1}{2}s$ indicates the value of the capacitance to be $\frac{1}{2}F$ in parallel with $Y_4(s) = \left(\frac{1}{s}\right)$ which is an inductor of $L = 1H$



Ex-2 $\quad Z(s) = \dfrac{s^4+10s^2+7}{s^3+2s}$, Realise the network.

soln Step-1
Here a pole is at infinity. indicating presence of an series inductor.

Step-2

$$s^3+2s \,\Big|\, s^4+10s^2+7 \,\Big|\, s$$
$$\underline{s^4+2s^2}$$
$$8s^2+7$$

$\therefore \quad Z(s) = s + \dfrac{8s^2+7}{s^3+2s} = Z_1(s) + Z_2(s)$

i.e. Inductor $(L) = 1H$

step-3

$$Y_2(s) = \frac{s^3+2s}{8s^2+7}$$

$Y_2(s)$, would have pole at infinity resulting eap parallel capacitor.

step-4

$$8s^2+7 \overline{\big)\, s^3+2s \,\big(} \frac{s}{8}$$
$$\underline{s^3+\frac{7}{8}s}$$
$$\frac{9}{8}s$$

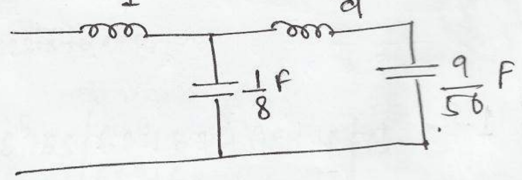
$$Y_2(s) = \frac{s}{8} + \frac{\frac{9}{8}s}{8s^2+7} = Y_3(s) + Y_4(s)$$

i.e Capacitor (C) $= \frac{1}{8}$ F.

step-4

$$Z_4(s) = \frac{8s^2+7}{\frac{9}{8}s} = \frac{64}{9}s + \frac{56}{9s}$$

i.e Inductor (L) $= \frac{64}{9}$ H & Capacitor (C) $= \frac{9}{56}$ F connected in series.



Ex-3

$$Z(s) = \frac{s^2+4s+40}{s(s+10)}$$, Realise the network.

sol^n

step-1

Here pole is at origin. Indicates the presence of series capacitor.

step-2

$$s^2+10s \,\big)\, s^2+4s+40 \,\big(\, \frac{4}{s}$$
$$\underline{s^2+4s+40}$$
$$s^2$$

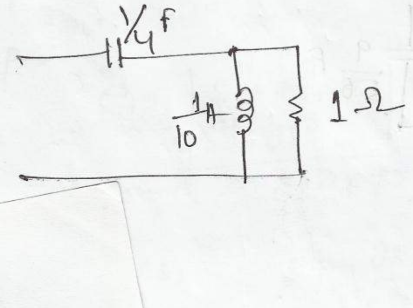
$$Z(s) = \frac{4}{s} + \frac{s^2}{s^2+10s} = Z_1(s) + Z_2(s)$$

So, $Z_1(s) = \frac{4}{s}$ indicating capacitor having $C_1 = \frac{1}{4} f$ in series with $Z_2(s)$.

step-3

$$Y_2(s) = \frac{s^2+10s}{s^2} = \frac{s+10}{s} = 1 + \frac{10}{s}$$

So, inductor parallel with resistor.



Ex $Z(s) = \frac{6s^3+5s^2+6s+4}{2s^3+2s}$, Realise the N/w.

Sol^n

$$Z(s) = \frac{6s^3+5s^2+6s+4}{s(2s^2+2s)}$$

step-1

$$Re\left[Z(j\omega)\right]_{s=j\omega} = Re\left[\frac{-j6\omega^3 - 5\omega^2 + 6j\omega + 4}{-j2\omega^3 - 2\omega^2}\right] \times \frac{(-2\omega^2 + j2\omega^2)}{(-2\omega^2 + j2\omega^2)}$$

$$= \frac{-5\omega^2+4}{-2\omega^2} = Re$$

$$= 12\omega^6 + 10\omega^4 - 12\omega^4 - 8\omega^2$$

derivative $= 72\omega^5 - 8\omega^3 - 16\omega^2 = 0$

$$\omega(72\omega^4 - 8\omega^2 - 16) = 0$$

$$\Rightarrow 72\omega^4 - 8\omega^2 = 16$$

$$\Rightarrow \omega^2(72\omega^2 - 8) = 16$$

$$\boxed{\omega = \pm 4}$$

As the min. real value is const, thus the 1st element is a resistance.

So.

$$2s^3+2s \overline{\left)\, 6s^3+5s^2+6s+4 \,\right(} \; 3$$
$$\underline{6s^3+6s}$$
$$5s^2+4 \,)\, 2s^3+2s$$

$$\therefore \; Z(s) = 3 + \frac{5s^2+4}{2s^3+2s} = Z_1(s) + Z_2(s)$$

Step-2

$$Z_2(s) = \frac{5s^2+4}{2s^3+2s} \qquad Y_2(s) = \frac{2s^3+2s}{5s^2+4}$$

As pole at $s=\infty$ indicating the presence of a parallel capacitor.

$$5s^2+4 \overline{)\,2s^3+2s\,}\left(\frac{2s}{5}\right.$$
$$\underline{2s^3+\frac{8}{5}s}$$
$$\frac{2}{5}s$$

$$Y_2(s) = \frac{2s}{5} + \frac{2s/5}{5s^2+4} = Y_3(s) + Y_4(s)$$

$$\therefore C = \frac{2}{5}\,f$$

Step-3

$$Z_4(s) = \frac{5s^2+4}{2s/5} = \frac{25s^2+20}{2s} =$$

Again pole at $s=\infty$ indicating a series inductor.

$$2s \overline{)\,25s^2+20\,}\left(\frac{25s}{2}\right.$$
$$\underline{25s^2}$$
$$20$$

$$Z_4(s) = \frac{25}{2}s + \frac{10}{s}$$

So, inductor $L = \frac{25}{8}$ f capacitor $C = \frac{1}{10}$ f

Ex $Y(s) = \frac{4s^2+6s}{s+1}$, Realize the network.

Soln

Step-1 - There pole exists at $s = \infty$, indicates that a parallel capacitor exists.

Step-2

$$s+1 \,\big)\, 4s^2+6s \,\big(\, 4s$$
$$\underline{4s^2+4s}$$
$$2s$$

$$Y(s) = 4s + \frac{2s}{s+1} = Y_1(s) + Y_2(s)$$

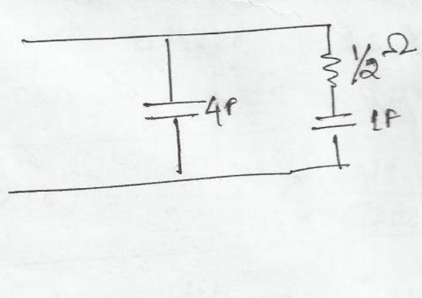
So, capacitor (C) = 4F

Step-2

$$Z_2(s) = \frac{s+1}{2s} = \frac{1}{2} + \frac{1}{s} = Z_3(s) + Z_4(s)$$

As, $Z_2(s)$ being having a pole at $s=0$, it is evident that there will be series capacitor.

So, $C = 1F$ & $R = \frac{1}{2}\,\Omega$



Ex $Y(s) = \frac{7s+5}{3s+9}$, Realize network

Sol^n Step-1

$$Re\left[Y(j\omega)\right] = Re\left[\frac{(7j\omega+5)}{(j3\omega+9)} \times \frac{(9-j3\omega)}{(9-j3\omega)}\right]$$

$$= Re\left[\frac{63j\omega + 21\omega^2 + 45 - 15j\omega}{81+9\omega^2}\right]$$

$$= \frac{21\omega^2+45}{81+9\omega^2}$$

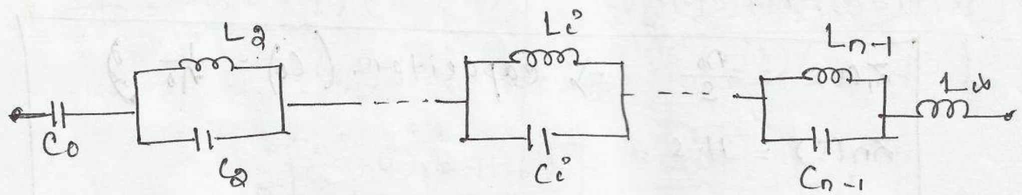
$$Min\left[Re\, Y(j\omega)\right] = \frac{5}{9}$$

So, it indicates the resistance of $\frac{5}{9}\,\Omega$ is connected in parallel as first element.

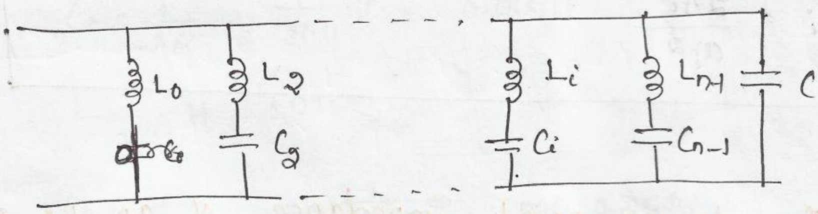
step-2

LC Network Synthesis

1) Foster's Canonic Form



[First form of Foster LC N/w (Impedance Form)]



2nd form of Foster LC N/w (Admittance form)

If $Z(s)$ being PR function having simple poles at $j\omega$ axis [$s = \pm j\omega_i$, $i = 1, 2, 3 \dots m$] as well as simple poles at $s=0$ & $s = \infty$, using partial fraction, $Z(s)$ can be expressed as

$$Z(s) = \frac{A_0}{s} + \sum_{i=2}^{n-1} \frac{2A_i s}{s^2 + \omega_i^2} + H \cdot s$$

when the constituent terms in $Z(s)$ can be interpreted as –

(i) $\frac{A_0}{s}$ results from a possible pole at $s=0$ (1st term)

(ii) $H \cdot s$ results from a possible pole at $s = \infty$ (last term)

(iii) $\frac{2A_i s}{s^2 + \omega_i^2}$ results from a pair of conjugate poles on $j\omega$ axis.

From eq^n (1)

$$Z(s) = \frac{A_0}{s} + \frac{2A_2 s}{s^2+\omega^2} + \cdots + \frac{2A_{n-1} s}{s^2+\omega_{n-1}^2} + H \cdot s$$

$$= Z_1(s) + Z_2(s) + \cdots + Z_n(s)$$

$Z_1(s) = \frac{A_0}{s}$ => capacitor (C_0) $= \frac{1}{A_0}$
$Z_n(s) = H \cdot s$
$C_i = \frac{1}{2A_i}$
$L_i = \frac{2A_i}{\omega_i^2}$

Ex The driving point impedance of an LC N/w is $Z(s) = 10 \frac{(s^2+4)(s^2+16)}{s(s^2+9)}$ Obtain the 1st form of foster Network.

Sol^n Here two poles exist at $\omega = 0$ & at $\omega = \infty$, Thus, the N/w consist of 1st & last element in the foster 1st form.

By taking the partial fraction expansion of $Z(s)$, we find

$$Z(s) = \frac{A_0}{s} + \frac{2A_2 s}{s^2+9^2} + H s$$

$$= \frac{A_0}{s} + \frac{A_2}{s+j3} + \frac{A_2^*}{s-j3} + Hs$$

where $A_0 = \left.\dfrac{10(s^2+4)(s^2+16)}{(s^2+9)}\right|_{s=0} = \dfrac{10\times4\times16}{9} = 71.11$

$A_2 = \left.\dfrac{10(s^2+4)(s^2+16)}{s(s-j3)}\right|_{s=-j3} = \dfrac{10[\{(-j3)^2+4\}\{((-j3)^2+16\}]}{(-j3)(-j3-j3)}$

$= 10\left[\dfrac{(-9+4)(-9+16)}{-18}\right] = \dfrac{350}{18} = 19.45$

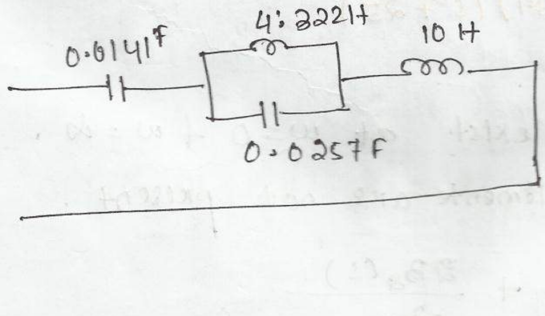
So,

$C_0 = \dfrac{1}{A_0} = \dfrac{1}{71.11}F = 0.01411F$

$L_\infty = H = 10H$

$C_2 = \dfrac{1}{2A_2} = \dfrac{1}{2\times19.45} = 0.0257F$

$L_2 = \dfrac{2A_2}{\omega_n^2} = \dfrac{2\times19.45}{(9)^2} = 4.322H$



Foster - II form

$$Y(s) = \frac{B_0}{s} + \frac{2B_2(s)}{s^2+\omega_2^2} + \cdots + \frac{2B_i(s)}{s^2+\omega_i^2} + \frac{2B_{n-1}(s)}{s^2+\omega_{n-1}^2} + H\cdot s$$

$$= Y_1(s) + Y_2(s) + \cdots + Y_n(s)$$

$Y_1(s) = \frac{B_0}{s} \Rightarrow L_0 = \frac{1}{B_0}\ H$

$Y_n(s) = H\cdot s \Rightarrow C_\infty = H\ F$

$L_i = \frac{1}{2B_i}$

$C_i = \frac{2B_i}{\omega_i^2}$

Ex Find the 2nd Foster form of the admittance function

$$Y(s) = \frac{s(s^2+9)}{10(s^2+4)(s^2+25)}$$

Sol^n

Here two zero exist at $\omega = 0$ & $\omega = \infty$.

Thees 1st & last elemente are not present.

$$Y(s) = \frac{2B_1 s}{s^2+4} + \frac{2B_2(s)}{s^2+25}$$

$$B_1 = \frac{1}{10}\left.\frac{s(s^2+9)}{(s-j2)(s^2+25)}\right|_{s=j2} = \frac{1}{10}\times\frac{10}{84} = \frac{1}{84}$$

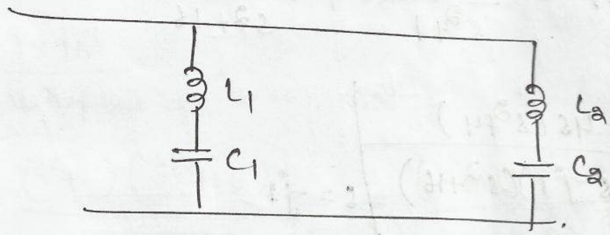
$$B_2 = \frac{1}{10}\left.\frac{s(s^2+9)}{(s^2+4)(s-j5)}\right|_{s=j5} = \frac{80}{10\times 210} = \frac{1}{26.25}$$

$$L_1 = \frac{1}{2B_1} = 42H$$

$$C_1 = \frac{2B_1}{\omega_1^2} = \frac{2 \times \frac{1}{84}}{2^2} = \frac{1}{168} F$$

$$L_2 = \frac{1}{2B_2} = 26.25/2 = 13.125H$$

$$C_2 = \frac{2B_2}{\omega_2^2} = \frac{2 \times 1/26.25}{5^2} = \frac{2}{656.25} F$$



Ex-3

Q

$$Z(s) = \frac{2(s^2+9)(s^2+16)}{s(s^2+4)}$$

Q

$$Z(s) = \frac{8(s^2+4)(s^2+25)}{s(s^2+16)}$$

Obtain the 1st & 2nd. ~~form~~ Foster form.

Ex $Z(s) = \dfrac{4\, s(s^2+4)}{(s^2+1)(s^2+16)}$

Obtain the foster forms of LC N/W realisation

Soln

Here two zero one at $w=0$ & one at $w=\infty$ are present. so there will be no end elements.

So, $$Z(s) = \frac{2A_2 s}{s^2+1} + \frac{2A_3 s}{s^2+16}$$

$$A_2 = \left.\frac{4s(s^2+4)}{(s-j1)(s^2+16)}\right|_{s=-j1}$$

$$= \frac{4(j1)(-1+4)}{-2j(-1+16)} = \frac{2}{5} = 0.4$$

$$A_3 = \left.\frac{4s(s^2+4)}{(s^2+1)(s-j4)}\right|_{s=-j4}$$

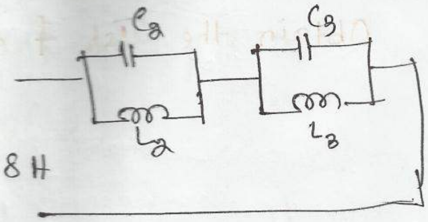
$$= \frac{4(-j4)(-16+4)}{(-16+1)\times -j8} = 1.6$$

$$C_2 = \frac{1}{2A_2} = \frac{1}{0.8}\,F$$

$$L_2 = \frac{2A_2}{\omega_2^2} = \frac{2\times 0.4}{12} = 0.8\,H$$

$$C_3 = \frac{1}{2A_3} = \frac{1}{2\times 16} = \frac{1}{3.2}\,F$$

$$L_3 = \frac{2A_2}{\omega_3^2} = \frac{3.2}{16} = 0.2\,H$$



Foster Form - 2

$$Y(s) = \frac{(s^2+1)(s^2+16)}{4s(s^2+4)}$$

Here the given function has two poles at $\omega=0$ & $\omega=\infty$ so, it contain the end elements.

$$Y(s) = \frac{B_0}{s} + \frac{B_2}{s+j2} + \frac{B_2^*}{s-j2} + H \cdot s$$

$$B_0 = \frac{1 \times 16}{4 \times 4} = 1$$

$$B_2 = \left.\frac{(s^2+1)(s^2+16)}{4s(s-j2)}\right|_{s=-j2} = \frac{(-4+1)(-4+16)}{-8j \times j4} = 1.125$$

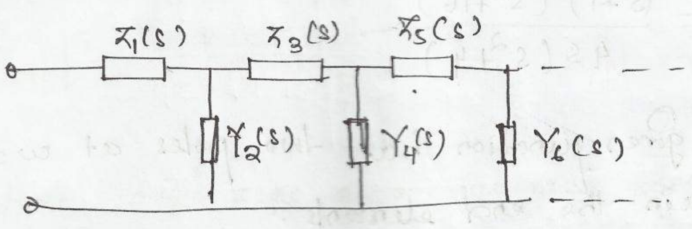
$$L_0 = \frac{1}{B_0} = 1H$$

$$C_\infty = H = 0.25F$$

$$L_2 = \frac{1}{2B_2} = \frac{1}{2 \times 1.125} = \frac{1}{2.25} H$$

$$C_2 = \frac{2B_2}{\omega_2^2} = \frac{2 \times 1.125}{4} = 0.5625F$$

Cauer Canonic Form



(Ladder Network)

The driving point impedance of this N/W may be represented in the form of continued fraction as

$$Z(s) = Z_1(s) + \cfrac{1}{Y_2(s) + \cfrac{1}{Z_3(s) + \cfrac{1}{Y_4(s) + \cfrac{1}{Z_5(s) + \cdots}}}}$$

First form of Cauer Network

Cauer proposed the first form to have L in series & C in shunt in the ladder pattern.

$$Z(s) = \frac{a_n s^n + a_{n-2} s^{n-2} + \cdots + a_0}{b_m s^m + b_{m-2} s^{m-2} + \cdots + b_1 s}$$

Cauer first form network can be realised by pole zero configuration of driving point function, where, for n>m, pole appears at $w = \infty$ which represents the first element to be a series inductor, and in the same expression zero appears with $w = 0$ indicating the last element to be an inductor.

On the other hand, for $m>n$, zero appears for $\omega=\infty$ indicating the first element to be a shunt capacitor and pole appears with $\omega=0$ indicating the last element to be capacitor.

(i) $\underset{s\to\infty}{Z(s)} \to \infty$ if $n>m$

$$Z_1(s) = L_1 s + \cfrac{1}{C_2 s + \cfrac{1}{L_3 s + \cfrac{1}{C_3 s + \cdots}}}$$

(ii) $\underset{s\to\infty}{Z(s)\to 0}$ if $n<m$

i.e. $L_1 = 0$

$$Z(s) = \cfrac{1}{C_2 s + \cfrac{1}{L_3 s + \cfrac{1}{C_4 s + \cdots}}}$$

Ex $Z(s) = \dfrac{s(s^2+4)(s^2+6)}{(s^2+1)(s^2+5)}$ Obtain the first form of Cauer network.

sol^n

$$Z(s) = \frac{s(s^4+10s^2+24)}{s^4+6s^2+5} = \frac{s^5+10s^3+24s}{s^4+6s^2+5}$$

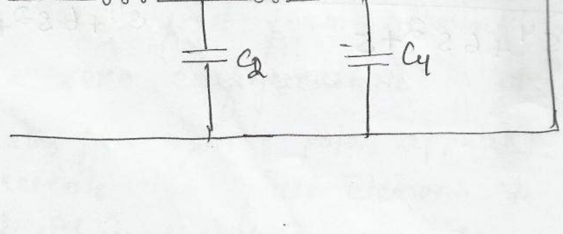
$$s^4+6s^2+5 \,\Big)\, s^5+10s^3+24s \,\Big(\, s$$
$$s^5+6s^3+5s$$
$$4s^3+19s \,\Big)\, s^4+6s^2+5 \,\Big(\, \frac{s}{4}$$
$$s^4+\frac{19s^2}{4}$$
$$\frac{5}{4}s^2+5 \,\Big)\, 4s^3+19s \,\Big(\, \frac{16}{5}s$$
$$4s^3+16s$$
$$3s \,\Big)\, \frac{5}{4}s^2+5 \,\Big(\, \frac{5}{12}s$$
$$\frac{5}{4}s^2$$
$$5 \,\Big)\, 3s \,\Big(\, \frac{3}{5}s$$
$$3s$$
$$\times$$

Here $Z(s) \to \infty$ with $s \to \infty$
also $Z(s) \to 0$ with $s \to 0$.
Thus for the function $Z(s)$ having $n > m$, the first element is L_1 & the last element is also an inductor.

$$Z(s) = s + \cfrac{1}{\frac{s}{4} + \cfrac{1}{\frac{16}{5}s + \cfrac{1}{\frac{5}{12}s + \cfrac{1}{\frac{3s}{5}}}}}$$

$L_1 = 1H$ $\quad L_3 = \frac{16}{5} H$ $\quad L_5 = \frac{3}{5} H$

$C_2 = \frac{1}{4} F$ $\quad C_4 = \frac{5}{12} F$



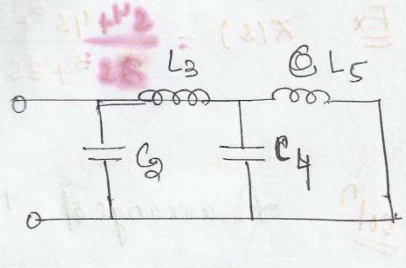
Q. Develop the cauer - 1 network for the given function

$$Z(s) = \frac{s^5+5s^3+3s}{s^4+3s^2+1}$$

Ex $Z(s) = \dfrac{6s^3+2s}{9s^4+4s^2+(1/16)}$. find the first form of Cauer network.

Sol^n Since the order of polynomial of denominator is higher than that of numerator, hence we first invert the function & then proceed with continued fraction. It is observed that zero is form at $s\to\infty$ indicates absence of first element. However, with $s\to 0$, $Z(s)\to 0$. ~~Since here~~ So the last element is inductor.

$$6s^3+2s \,\big)\, 9s^4+4s^2+\tfrac{1}{16} \,\big(\, \tfrac{3}{2}s$$
$$9s^4+3s^2$$
$$s^2+\tfrac{1}{16} \,\big)\, 6s^3+2s \,\big(\, 6s$$
$$6s^3+s$$
$$s \,\big)\, s^2+\tfrac{1}{16} \,\big(\, s$$
$$s^2$$
$$\tfrac{1}{16} \,\big)\, s \,\big(\, 16s$$
$$s$$
$$\times$$



$$Z(s) = \cfrac{1}{\cfrac{3s}{2} + \cfrac{1}{6s + \cfrac{1}{s + \cfrac{1}{6s}}}}$$

$C_2 = \frac{3}{2}F$ $\quad L_5 = 6H$

$L_3 = 6H$

$C_4 = 1F$

Cauer Form-II

Cauer proposed the 2nd form to have C is series & L in shunt in the ladder pattern.

(i) If $Z(s) \to \infty$ with $s \to 0$, a pole appears in $Z(s)$ hence C_1 must have some definite value.

$$Z(s) = \frac{1}{C_1 s} + \cfrac{1}{\cfrac{1}{L_2 s} + \cfrac{1}{\cfrac{1}{C_3 s} + \cfrac{1}{\cfrac{1}{L_4 s} + \cdots}}}$$

(ii) If $Z(s) \to 0$ as $s \to 0$, C_1 must be zero.

(iii) If $Z(s) \to 0$ as $s \to \infty$, the last element is capacitor.

(iv) If $Z(s) \to \infty$ as $s \to \infty$, the last element must be inductor.

Ex $Z(s) = \dfrac{s^4 + 4s^2 + 3}{2s^3 + 3s}$, Find the 2nd form of Cauer network.

Sol^n Rearrange the given function

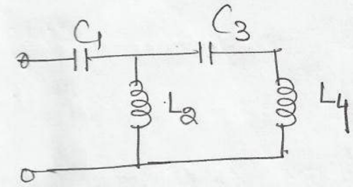
$$Z(s) = \frac{3 + 4s^2 + s^4}{3s + 2s^3}$$

$$3s+2s^3 \,\big)\, 3+4s^2+s^4 \,\big(\, \frac{1}{s}$$
$$3+2s^2$$
$$2s^2+s^4 \,\big)\, 3s+2s^3 \,\big(\, \frac{3}{2s}$$
$$3s+\frac{3}{2}s^3$$
$$\frac{s^3}{2} \,\big)\, 2s^2+s^4 \,\big(\, 4/s$$
$$2s^2$$
$$s^4 \,\big)\, \frac{s^3}{2} \,\big(\, \frac{1}{2s}$$
$$\frac{s^3}{2}$$
$$\times$$

With $\lim_{s\to 0} z(s) \to \infty$, then the first element is capacitor.

& $\lim_{s\to\infty} z(s) \to \infty$, then the last term to be inductor.

$$z(s) = \frac{1}{s} + \cfrac{1}{\cfrac{3}{2s} + \cfrac{1}{\cfrac{4}{s} + \cfrac{1}{\cfrac{1}{2s}}}}$$



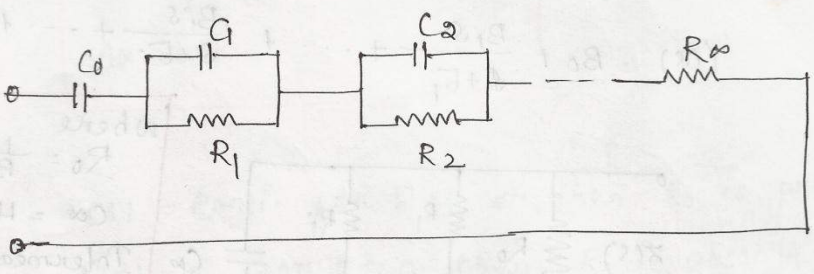
$C_1 = 1F$ $\qquad$ $C_3 = \frac{1}{4}F$

$L_2 = \frac{2}{3}H$ $\qquad$ $L_4 = 2H$

Q $z(s) = \dfrac{[illegible] + 12s^3 + 4s}{3s^4 + 10s^2 + 2}$, synthesize the funct^n using Cauer-2 N/W.

RC Network Synthesis by Foster Form

The RC N/W in Foster first form can be represented by



The impedance of a RC N/W in Foster first form can be represented as

$$Z(s) = \frac{A_0}{s} + \frac{A_1}{s+\sigma_1} + \frac{A_2}{s+\sigma_2} + \cdots + A_\infty$$

where $C_0 = \frac{1}{A_0}$

$C_1 = \frac{1}{A_1}$, $C_2 = \frac{1}{A_2}$ ...

$\sigma_1 = \frac{1}{R_1 C_1} \Rightarrow R_1 = \frac{A_1}{\sigma_1}$.

$\sigma_2 = \frac{1}{R_2 C_2} \Rightarrow R_2 = \frac{A_2}{\sigma_2}$

$R_\infty = A_\infty$.

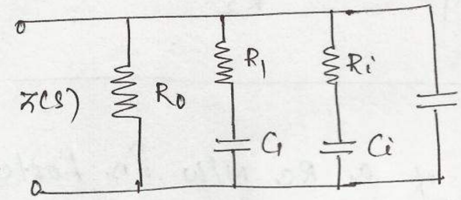
→ If $Z(s) = \infty$ for frequency $\sigma = 0$, C_0 is present, but if $Z(s)$ is a constant for $\sigma = 0$, it is evident that C_0 is absent.

→ If $Z(s)$ is a constant for $\sigma = \infty$, then last term is present. On the other hand, if $Z(s) = 0$ for $\sigma = \infty$, then last term R_∞ is missing.

Foster 2nd Form

The driving point addmittance of a RC network is Foster 2nd form can be represented as

$$Y(s) = B_0 + \frac{B_1 s}{s+\sigma_1} + \cdots + \frac{B_i s}{s+\sigma_i} + \cdots + Hs$$



where
$R_0 = \frac{1}{B_0}$
$C_\infty = H$
Intermeadte terms $Y_i(s)$ is $Y_i(s) = \frac{1}{R_i + \frac{1}{C_i s}}$

⇒ If $Z(s)$ = constant for $\sigma = 0$, first element is present
$Z(s) = \infty$ for $\sigma = 0$, first element is absent

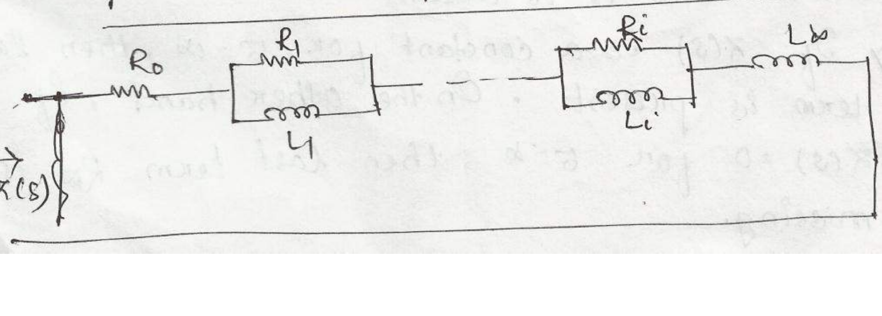
⇒ If $Z(s) = 0$ for $\sigma = \infty$, last element is present
$Z(s)$ = const. for $\sigma = \infty$, last element is absent.

RL Network Synthesis by Foster Form

Foster-I

The driving point impedance of a RL network in Foster first form

$$Z(s) = A_0 + \frac{A_1 s}{s+\sigma_1} + \cdots + \frac{A_i s}{s+\sigma_i} + A_\infty s$$



where

$A_0 = R_0$

$A_1 = R_1$, ... $A_i = R_i$

$L_1 = \frac{A_1}{\sigma_1}$, $L_i = \frac{A_i}{\sigma_i}$

$A_\infty = L_\infty$

⇒ If $Z(s)$ = const. for $\sigma = 0$, then R_0 is present

$Z(s) = 0$. for $\sigma = 0$ then R_0 is absent

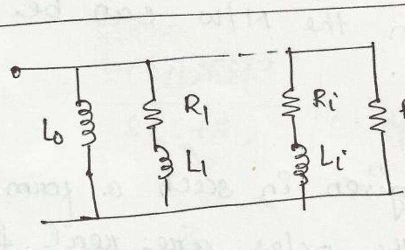
⇒ If $Z(s) = \infty$ for $\sigma = \infty$ then L_∞ is present

$Z(s)$ = const. for $\sigma = \infty$, then L_∞ is absent.

Foster-II

The driving point admittance function of RL n/w for Foster 2nd form

$$Y(s) = \frac{B_0}{s} + \frac{B_1}{s+\sigma_1} + \cdots + \frac{B_i}{s+\sigma_i} + H$$



$L_0 = \frac{1}{B_0}$

$R_\infty = H$

$Y_n(s) = \frac{1}{R_n + sL_n}$

⇒ If $Z(s) = 0$, for $\sigma = 0$ then L_0 present

$Z(s)$ = const , for $\sigma = 0$ then L_0 absent

⇒ If $Z(s)$ = const , for $\sigma = \infty$ then R_∞ present

$Z(s) = \infty$ for $\sigma = \infty$, then R_∞ absent.

Identification of Foster form of R-L / R-C N/W from any given function

1) If the driving point impedance function can be represented by

$$Z(s) = A_0 + \frac{A_i s}{s+\sigma_i} + \cdots + Hs$$

and if the residues at the poles of $Z(s)$ must be real and negative though the residues of $\frac{Z(s)}{s}$ are real and positive, then the N/W can be realize is Foster first form.

2) If the driving point admittance function can be represented by

$$Y(s) = \frac{B_0}{s} + \cdots + \frac{B_i}{s+\sigma_i} + \cdots + H$$

and the residues at the poles of the function are real and +ve, then the N/W can be realised in Foster second form.

3) Again if $Z(s)$ be given in such a form such that the residues at the poles are real & +ve

& $Z(s) = Z_1(s) + \cdots + Z_i(s) + \cdots + Z_n(s)$

$$\Rightarrow Z(s) = \frac{A_0}{s} + \cdots + \frac{A_i}{s+\sigma_i} + \cdots H$$

the N/W realisation is done in Foster first form of RC N/W

4) Similarly if $Y(s)$ is given in the form

$$Y(s) = B_0 + \cdots + \frac{B_i s}{s+\sigma_i} + \cdots + Hs$$

with the residues as real and -ve then the N/W realisation is done in foster's 2nd form.

Ex-1

$$Z(s) = \frac{2(s+1)(s+3)}{(s+2)(s+4)}$$ Find R-L representation of foster first form of N/W.

Sol^n

$$Z(s) = 2 - \frac{1}{s+2} - \frac{3}{s+4}$$, coefficients are -ve,

So, we take $Z(s)/s$.

$$\frac{Z(s)}{s} = \frac{2(s+1)(s+3)}{s(s+2)(s+4)} = \frac{K_1}{s} + \frac{K_2}{s+2} + \frac{K_3}{s+4}$$

$$K_1 = \left.\frac{2(s+1)(s+3)}{(s+2)(s+4)}\right|_{s=0} = \frac{2\times1\times3}{2\times4} = \frac{3}{4}$$

$$K_2 = \left.\frac{2(s+1)(s+3)}{s(s+4)}\right|_{s=-2} = \frac{2\times-1\times1}{-2\times2} = \frac{1}{2}$$

$$K_3 = \left.\frac{2(s+1)(s+3)}{s(s+2)}\right|_{s=-4} = \frac{2\times-3\times-1}{-4\times-2} = \frac{3}{4}$$

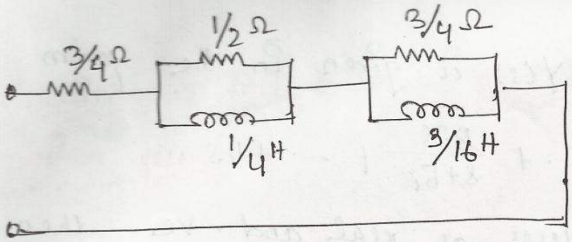
So

$$\frac{Z(s)}{s} = \frac{3}{4s} + \frac{1/2}{s+2} + \frac{3/4}{s+4}$$

$$\Rightarrow Z(s) = \frac{3}{4} + \frac{(1/2)s}{(s+2)} + \frac{(3/4)s}{s+4}$$

So, $R_0 = \frac{3}{4}\Omega$, $R_1 = \frac{1}{2}\Omega$, $R_2 = \frac{3}{4}\Omega$

$L_1 = \frac{A_1}{\sigma_1} = \frac{1}{4}H$, $L_2 = \frac{\frac{3}{4}}{4} = \frac{3}{16}H$



Ex-2

$$Y(s) = \frac{(s+4)(s+6)}{(s+3)(s+5)}$$. Find R-L n/w following Forster form of realisation.

Sol^n

$$Y(s) = \frac{(s+4)(s+6)}{(s+3)(s+5)}$$

Here for s = 0, Y(s) is const. so, L_0 is absent.

~~$$\frac{Y(s)}{s} = \frac{(s+4)(s+6)}{s(s+3)(s+5)} = \frac{K_1}{s} + \frac{K_2}{s+3} + \frac{K_3}{s+5}$$~~

~~$$K_1 = \frac{(s+4)(s+6)}{(s+3)(s+5)}\Bigg|_{s=0} = \frac{4\times 6}{3\times 5} = \frac{8}{5}$$~~

$$Y(s) = \frac{s^2+10s+24}{s^2+8s+15} = 1 + \frac{2s+9}{s^2+8s+15} = 1 + \frac{2s+9}{(s+3)(s+5)}$$

$$= 1 + \frac{A}{s+3} + \frac{B}{s+5}$$

$$A = \frac{2s+9}{s+5}\Bigg|_{s=-3} = \frac{-6+9}{2} = \frac{3}{2}$$

$$B = \frac{2s+9}{s+3}\Bigg|_{s=-5} = \frac{-10+9}{-2} = \frac{1}{2}$$

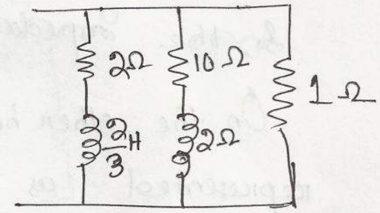
$$Y(s) = 1 + \frac{(3/2)}{s+3} + \frac{(1/2)}{s+5} = 1 + \frac{3}{2(s+3)} + \frac{1}{2(s+5)}$$

$$\Rightarrow \boxed{Y(s) = 1 + \frac{1}{\frac{2s}{3}+2} + \frac{1}{2s+10}}$$

So, $R_\infty = 1\,\Omega$

$R_1 = 2\Omega$, $L_1 = \frac{2}{3}H$

$R_2 = 10\,\Omega$, $L_2 = 2\,\Omega$



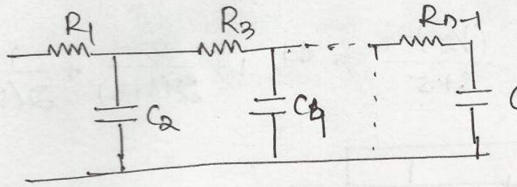
Cauer Form of Synthesis

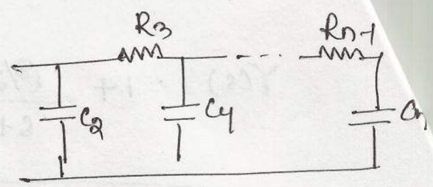
RC Network

If the driving point impedance function is represented as

$$Z(s) = R_1 + \cfrac{1}{C_2 s + \cfrac{1}{R_3 + \cfrac{1}{C_4 s + \cdots}}}$$

the 1st form of Cauer N/W can be realized. However, in the N/W if $Z(s)$ is zero at $s \to \infty$, the 1st element is shunt capacitor while for $Z(s)$ a const. for $s \to \infty$, the 1st element is series resistor R_1. Similarly for $Z(s)$ const at $s \to 0$, the last element is resistance.





(1st element C_2)

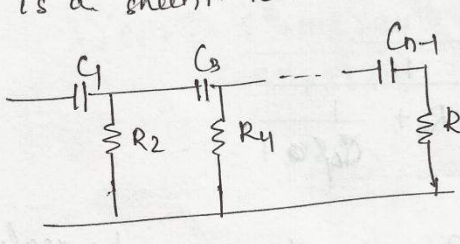
(1st element R_1)

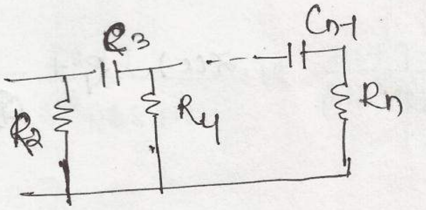
So, the impedance.

On the other hand, if the continued fraction is represented as

$$z(s) = \frac{1}{C_1 s} + \cfrac{1}{\cfrac{1}{R_2} + \cfrac{1}{\cfrac{1}{C_3 s} + \cfrac{1}{\cfrac{1}{R_4} + \cdots}}}$$

and the value $z(s)$ is infinity at $s \to 0$, then the N/w realisation is in the 2nd cauer form, in which the 1st element is a series capacitor while $z(s) \to$ const at $s \to 0$ then the 1st element is a shunt resistor.





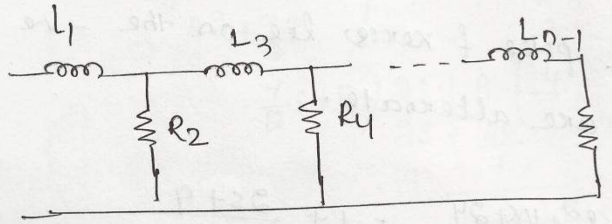
(1st element is R_2)

(1st element is C_1)

RL Network

In the Cauer 1st form

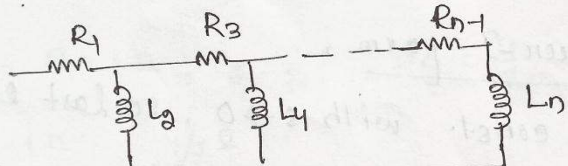
$$Z(s) = sL_1 + \cfrac{1}{\cfrac{1}{R_2} + \cfrac{1}{sL_3 + \cfrac{1}{\cfrac{1}{R_4} + \cdots}}}$$



If $Z(s) \to \infty$ for $s \to \infty$, the 1st element L_1 exists
but if $Z(s)$ is const. for $s \to \infty$ the 1st element absent.

In the Cauer 2nd form

$$Z(s) = R_1 + \cfrac{1}{\cfrac{1}{sL_2} + \cfrac{1}{R_3 + \cfrac{1}{\cfrac{1}{sL_4} + \cfrac{1}{R_5 + \cdots}}}}$$



If $Z(s)$ is const. at $s \to \infty$, the 1st element is R_1.
& if $Z(s) \to 0$ at $s \to 0$, the last element is L_n
& if $Z(s)$ is const. at $s \to 0$, the last element is R_n.

Ex $Z(s) = \frac{(s+4)(s+6)}{(s+3)(s+5)}$. Diagnose whether the following impedance funct^n represents a RL or RC n/w and find it 1st cauer form.

Sol^n

Here the poles & zeros lie on the -ve real axis and they are alternate.

$$Z(s) = \frac{s^2+10s+24}{s^2+8s+15} = 1 + \frac{2s+9}{(s+3)(s+5)}$$

$$= 1 + \frac{A}{s+3} + \frac{B}{s+5}$$

$A = \frac{3}{2}$ (real & +ve)

$B = 1/2$ (real & +ve)

As the residues are +ve & real, so the given funct^n $Z(s)$ is RC impedance function.

For cauer-I form,

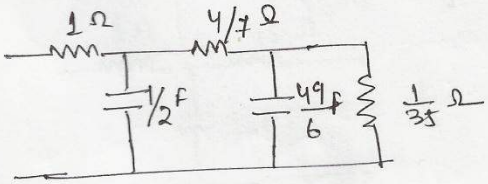
$Z(s)$ = a const. with $s \to 0$, so last element is resistance, again $Z(s) = 1$ with $s \to \infty$. so the 1st element is also a resistor.

$$Z(s) = \frac{s^2+10s+24}{s^2+8s+15}$$

$$
\begin{array}{l}
s^2+8s+15 \,\big)\, s^2+10s+24 \,\big(\, 1 \;\text{— } R_1 \\
\qquad\qquad\quad \underline{s^2+8s+15} \\
\qquad\qquad\quad 2s+9 \,\big)\, s^2+8s+15 \,\big|\, \frac{s}{2} \;\text{—} \\
\qquad\qquad\qquad\qquad \underline{s^2+\frac{9}{2}s} \\
\qquad\qquad\qquad\quad \frac{7}{2}s+15 \,\big)\, 2s+9 \,\big(\, \frac{4}{7} \\
\qquad\qquad\qquad\qquad\qquad \underline{2s+\frac{60}{7}} \\
\qquad\qquad\qquad\qquad\qquad \frac{3}{7} \,\big)\, \frac{7}{2}s+15 \,\big(\, \frac{49}{6}s \\
\qquad\qquad\qquad\qquad\qquad\qquad \underline{\frac{7}{2}s} \\
\qquad\qquad\qquad\qquad\qquad\qquad 15 \,\big)\, \frac{3}{7} \,\big(\, \frac{3}{7} \times \frac{1}{15} = \frac{1}{35} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad \underline{\frac{3}{7}} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad 0
\end{array}
$$

$$Z(s) = 1 + \cfrac{1}{\frac{1}{2}s + \cfrac{1}{\frac{4}{7} + \cfrac{1}{\frac{49}{6}s + \cfrac{1}{\frac{1}{35}}}}}$$

Here $R_1 = 1\,\Omega$, $C_2 = \frac{1}{2}F$, $R_3 = \frac{4}{7}\,\Omega$, $C_4 = \frac{49}{6}F$, $R_5 = \frac{1}{35}\,\Omega$



Notes

Case-I – Comparing $Z(s)$ value at $s \to \infty$ and $Z(s)$ value at $s \to 0$ and if $Z_s(\infty) < Z_s(0)$, it is an impedance function that can be of Cauer first form of RC N/W.

On the other hand, if the funct^n represents a zero at $s \to \infty$, it is the admittance function $Y(s)$ and it can be represented also by Cauer 1st form of RC N/W.

Case-II

Comparing $Z(s)$ value at $s \to \infty$ and $Z(s)$ value at $s \to 0$ if $Z_s(\infty) > Z_s(0)$, realisation is possible by Cauer 2nd form of RC N/W and this time the N/W function is the impedance function $Z(s)$.

On the other hand, if the function represents a constant value at $s \to 0$ and the given function is admittance then realisation is possible by Cauer 2nd form of RC N/W.

Ex $Z(s) = \dfrac{(s+3)(s+5)}{s(s+4)}$. Find the Cauer form of RC N/w.

Soln

Here $Z(s) = 0$ for $s \to \infty$ & $Z(s) = \infty$ for $s \to 0$.

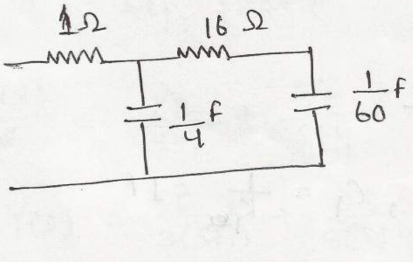
Since $Z_s(0) > Z_s(\infty)$, first form of RC N/w is to realised.

$$Z(s) = \frac{s^2 + 8s + 15}{s^2 + 4s}$$, $Z(s) \to \infty$ at $s \to 0$
so, the last element is capacitor

$s^2+4s \,)\, s^2+8s+15 \,(\, 1$
$\quad s^2+4s$
$\quad 4s+15 \,)\, s^2+4s \,(\, \frac{s}{4}$
$\quad\quad s^2+\frac{15}{4}s$
$\quad\quad \frac{s}{4} \,)\, 4s+15 \,(\, 16$
$\quad\quad\quad 4s$
$\quad\quad\quad 15 \,)\, \frac{s}{4} \,(\, \frac{s}{60}$
$\quad\quad\quad\quad \frac{s}{4}$
$\quad\quad\quad\quad \times$

$$Z(s) = 1 + \cfrac{1}{\cfrac{s}{4} + \cfrac{1}{16 + \cfrac{1}{\cfrac{s}{60}}}}$$

$R_1 = 1\,\Omega$, $C_2 = \frac{1}{4}\,F$, $R_3 = 16\,\Omega$, $C_4 = \frac{1}{60}\,F$



Ex $Z(s) = \frac{s^2+5s+4}{s^2+2s}$. Express it in both the foster forms.

sol^n

$$Z(s) = \frac{(s+1)(s+4)}{s(s+2)}$$

$$Z(s) = 1 + \frac{3s+4}{s^2+2s} = 1 + \frac{3s+4}{s(s+2)}$$

$$\Rightarrow Z(s) = 1 + \frac{K_0}{s} + \frac{K_2}{s+2}$$

$$K_0 = \left.\frac{3s+4}{s+2}\right|_{s=0} = 2$$

$$K_2 = \left.\frac{3s+4}{s}\right|_{s=-2} = \frac{-6+4}{-2} = 1$$

$$\boxed{Z(s) = 1 + \frac{2}{s} + \frac{1}{s+2}}$$

Here the residues are +ve and real & zeros are at −1, −4 while poles are at 0, −2, thus the poles and zeros alternate in the −ve real axis. Thus the given function can be realised as a RC N/w in the 1st form of foster N/w.

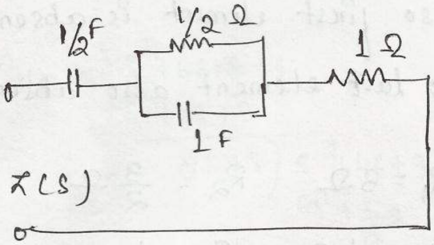
1st form represented as

$$Z(s) = \frac{2}{s} + \frac{1}{s+2} + 1$$

$$C_0 = \frac{1}{K_0} = \frac{1}{2} F \quad , \quad C_1 = \frac{1}{K_1} = 1 F$$

$$R_1 = \frac{K_1}{\sigma_1} = \frac{1}{2} \Omega$$

$$R_\infty = 1 \Omega$$



To obtain foster 2nd form

$$Y(s) = \frac{s^2+2s}{s^2+5s+4} = 1 - \frac{3s+4}{(s+1)(s+4)}$$

$$\Rightarrow Y(s) = 1 - \frac{K_1}{s+1} - \frac{K_2}{s+4}$$

Since negative co-efficients appear hence, we take $\frac{Y(s)}{s}$

$$\frac{Y(s)}{s} = \frac{(s^2+2s)}{s(s^2+5s+4)} = \frac{s(s+2)}{s(s+1)(s+4)} = \frac{s+2}{(s+1)(s+4)}$$

$$= \frac{K_1}{s+1} + \frac{K_2}{s+4}$$

$$K_1 = \left.\frac{s+2}{s+4}\right|_{s=-1} = \frac{1}{3}$$

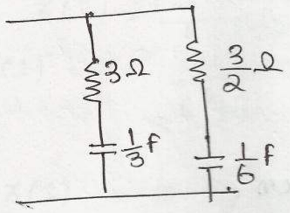
$$K_2 = \left.\frac{s+2}{s+1}\right|_{s=-4} = \frac{-2}{-3} = \frac{2}{3}$$

So, $$\frac{Y(s)}{s} = \frac{1/3}{s+1} + \frac{2/3}{s+4}$$

$$\Rightarrow Y(s) = \frac{s}{3(s+1)} + \frac{2s}{3(s+4)} = \frac{s}{3s+3} + \frac{s}{\frac{3}{2}s+6}$$

$$= \frac{1}{3+\frac{3}{s}} + \frac{1}{\frac{3}{2}+\frac{6}{s}} = \frac{1}{3+\frac{1}{(\frac{1}{3})s}} + \frac{1}{\frac{3}{2}+\frac{1}{(\frac{1}{6})s}}$$

Here $Z(s) = \infty$ at $\sigma \to 0$, so first element is absent
again $Z(s) = 1$ at $\sigma \to \infty$, so last element also absent



$R_1 = 3\Omega$ $R_2 = \frac{3}{2}\Omega$

$C_1 = \frac{1}{3}F$ $C_2 = \frac{1}{6}F$

Ex. $F(s) = \dfrac{s^2+6s+8}{s^2+4s+3}$, show that the function can be realised in both the Cauer RL & RC forms.

Sol^n

$$F(s) = \frac{s^2+6s+8}{s^2+4s+3} = \frac{(s+2)(s+4)}{(s+1)(s+3)}$$

Since the poles & zeros alternate on the -ve real axis and in both the case $F_s(0) > F_s(\infty)$ means $F(s)$ at $s \to 0$ greater than $F(s)$ at $s \to \infty$, hence both the RC & RL first cauer forms are realisable.

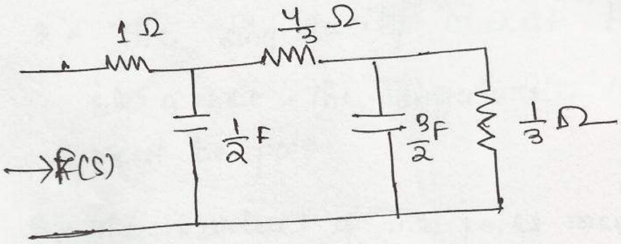
RC N/W

$F(s) \Rightarrow$ is a const at $s \to \infty$, 1st element is R_1.
& $F(s)$ is a const at $s \to 0$, last element is R_∞.

$s^2+4s+3 \overline{)\, s^2+6s+8 \,}(1$
s^2+4s+3
$2s+5 \overline{)\, s^2+4s+3 \,}(\frac{s}{2}$
$s^2+\frac{5}{2}s$
$\frac{3}{2}s+3 \overline{)\, 2s+5 \,}(\frac{4}{3}$
$2s+4$
$1 \overline{)\, \frac{3}{2}s+3 \,}(\frac{3}{2}s$
$\frac{3}{2}s$
$3 \overline{)\, 1 \,}(\frac{1}{3}$
1
$\times$

$$F(s) = 1 + \cfrac{1}{\frac{s}{2} + \cfrac{1}{\frac{4}{3} + \cfrac{1}{\frac{3}{2}s + \cfrac{1}{\frac{1}{3}}}}}$$

$R_1 = 1\Omega$, $C_2 = \frac{1}{2}F$, $R_3 = \frac{4}{3}\Omega$, $C_4 = \frac{3}{2}F$ & $R_5 = \frac{1}{3}\Omega$



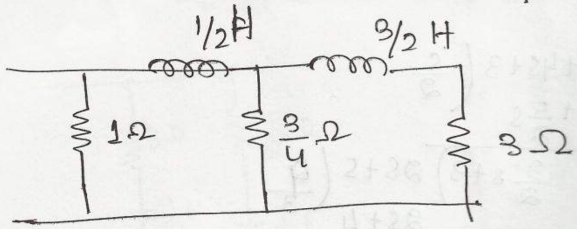
RL N/W

F(s) is a const. at $s \to \infty$, the 1st element is absent

F(s) is const at $s \to 0$, the last element is resistor.

$$F(s) = \cfrac{1}{1 + \cfrac{1}{\frac{s}{2} + \cfrac{1}{\frac{4}{3} + \cfrac{1}{\frac{3}{8}s + \cfrac{1}{\frac{1}{3}}}}}}$$

$L_1 = 0$, $R_2 = 1\Omega$, $L_3 = \frac{1}{2}H$, $R_4 = \frac{3}{4}\Omega$, $L_5 = \frac{3}{8}$ & $R_6 = 3\Omega$



Note

Properties of RC driving point impedance

1) Poles & zeros lie on -ve real axis.

2) Poles & zeros alternate on -ve real axis.

3) The singularity nearest to (or at) the origin must be a pole whereas the singularity nearest (or at) $\sigma = \infty$ must be zero.

4) The residues of the poles must be real & +ve.

Properties of RL driving point impedance

1) Poles & zeros of the RL impedance functions are located on the -ve real axis and are alternate.

2) The singularity nearest to (or at) the origin is a zero. The singularity nearest to (or at) $s=\infty$ must be pole.

3) The residues of the poles must be real & -ve for $Z(s)$ [for $\frac{Z(s)}{s}$, the residues are +ve & real]

Imp

Properties of RC driving point impedance

1) Poles & zeros lie on −ve real axis.

2) Poles & zeros alternate on −ve real axis.

3) The singularity nearest to (or at) the origin must be a pole whereas the singularity nearest (or at) $\sigma = \infty$ must be zero.

4) The residues of the poles must be real & +ve.

Properties of RL driving point impedance

1) Poles & zeros of the RL impedance functions are located on the −ve real axis and are alternate.

2) The singularity nearest to (or at) the origin is a zero. The singularity nearest to (or at) $s=\infty$ must be pole.

3) The residues of the poles must be real & −ve for $Z(s)$ [for $\frac{Z(s)}{s}$, the residues are +ve & real]

Active Filters

An electric filter is a four terminal frequency-selective network designed generally with reactive elements to transmit freely a specified band of frequency and block or attenuate signals of frequency outside this band.

→ The band of frequency transmitted through the filter is called the Pass-band.

→ The band of frequency which is attenuated by the filter is called the ~~sot~~ stop-band.

Classification -

(i) Analog or Digital Filters
(ii) Active or Passive Filters

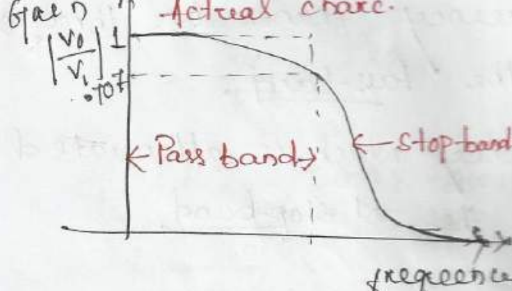
Analog filters are designed to process analog signals while digital filters process analog signals using digital techniques.

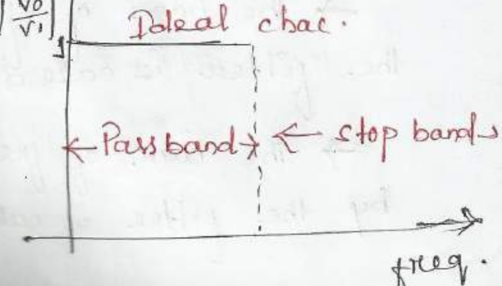
Passive filters consists of passive elements i.e. R, L & C. On the other hand, active filters consists of active components such as op-amp, transistors, in addition to R & C.

Types of Active Filters

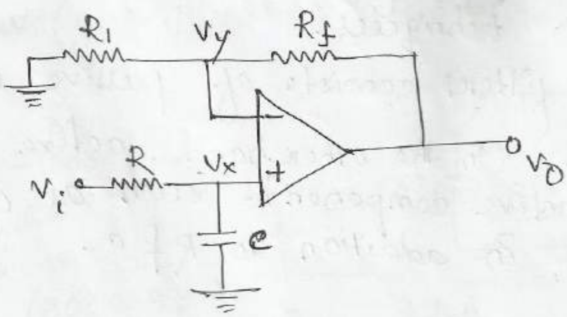
1. Low-Pass Filter :—

It is a circuit that has a constant output (or gain) from zero to a cut-off frequency, f_c and attenuation of all frequencies above f_c.





The filtering is done by the RC network, and the op-amp is used as a unity-gain amplifier. The resistor $R_f = R$.



$$v_x = \frac{X_C}{R + X_C} v_i = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \cdot v_i = \frac{\frac{1}{j2\pi f C}}{R + \frac{1}{j2\pi f C}}$$

$$\boxed{v_x = \frac{v_i}{1 + j2\pi f RC}}$$

$$\boxed{v_y = \frac{R_1}{R_1 + R_f} v_0}$$

Since op-amp gain is infinite

$$v_x = v_y$$

$$\Rightarrow \frac{v_i}{1 + j2\pi f RC} = \frac{R_1}{R_1 + R_f} v_0$$

$$\Rightarrow \frac{v_0}{v_i} = \frac{R(1 + R_f/R_1)}{1 + j2\pi f RC}$$

$$\Rightarrow \boxed{\frac{v_0}{v_i} = \frac{A_F}{1 + j(f/f_c)}} = A_{CL}$$

where $A_F = \left(1 + \frac{R_f}{R_1}\right)$ = Pass-band gain of the filter

f = frequency of the input signal

$f_c = \frac{1}{2\pi RC}$ = cutoff-frequency of the filter

A_{CL} = closed-loop gain of the filter as a function of frequency.

$$|A_{CL}| = \left|\frac{v_0}{v_i}\right| = \frac{A_F}{\sqrt{1 + (f/f_c)^2}} = \frac{A_F}{\sqrt{1 + \omega^2 R^2 C^2}}$$

$$\phi = -\tan^{-1}(f/f_c) = -\tan^{-1}(\omega RC)$$

Operation of the filter

The operation of the LPF can be verified from the gain magnitude equation as follows:

(i) At very low frequencies, i.e $f << f_c$

$$|A_{CL}| \cong A_F$$

(ii) At $f = f_c$, $|A_{CL}| = \frac{A_F}{\sqrt{2}} = 0.707 A_F$

$$= -3dB$$

$$\phi = 45^\circ$$

(iii) At $f > f_c$, $|A_{CL}| < A_F$

Thus, the filter has a constant gain of A_F from 0 Hz to the cut-off frequency f_c. At f_c, the gain is $0.707 A_F$ and after f_c, it decreases at a constant rate with an increase in frequency.

Filter Design

A LPF can be designed by implementing the following steps —

1) A value of the cut-off frequency ω_c (or f_c) is chosen.

2) A value of the capacitance C is selected, usually between 0.001 & 0.1 µF.

3. The values of the resistance R is calculated from the relation

$$R(\text{in } \Omega) = \frac{1}{\omega_c C} = \frac{1}{2\pi f_c C}$$

Q (a) Design a LPF active filter at a cutoff freq. of 1KHz with a passband gain of 2. Using the frequency scaling technique, convert this filter to a LPF of cutoff frequency 1.6 KHz

(b) Plot the frequency response of this new LPF.

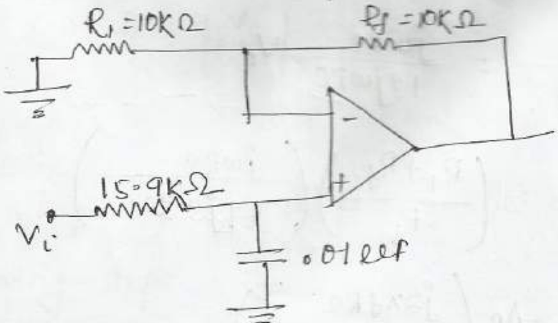
Sol^n

$f_c = 1KHz$, $A_F = 2$

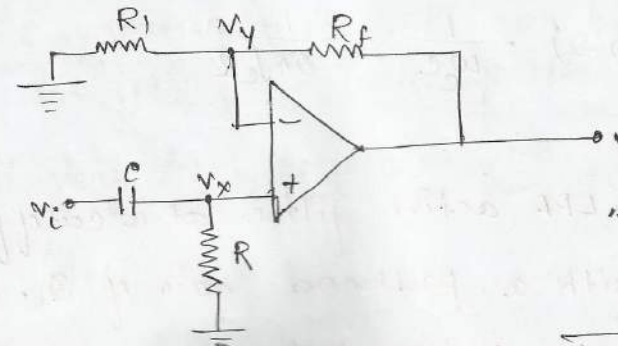
Let $C = 0.01 \mu F$

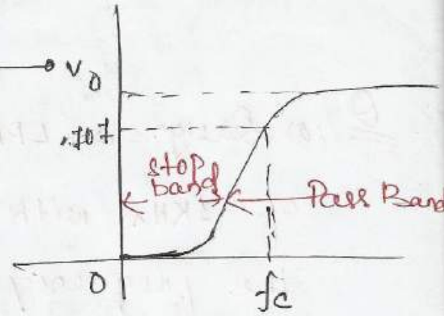
$$R = \frac{1}{2\pi f_c C} = \frac{1}{2\pi \times 10^3 \times .01 \times 10^{-6}} = 15.9 K\Omega$$

$$A_F = 2 = 1 + \frac{R_f}{R_1} \Rightarrow R_f = R_1 = 10K\Omega$$



High-Pass Active Filter





$$v_x = \frac{R}{R + X_C} v_i = \frac{R}{R + \frac{1}{j\omega C}} v_i$$

$$\Rightarrow \boxed{v_x = \frac{j\omega RC}{1 + j\omega RC} v_i}$$

$$\boxed{v_y = \frac{R_1}{R_1 + R_f} v_o}$$

Since op-amp gain is infinite

$$v_x = v_y$$

$$\Rightarrow \frac{v_o R_1}{R_1 + R_f} = \frac{j\omega RC}{1 + j\omega RC} v_i$$

$$\Rightarrow \frac{v_o}{v_i} = \left(\frac{R_1 + R_f}{R_1}\right)\left(\frac{j\omega RC}{1 + j\omega RC}\right)$$

$$\Rightarrow \frac{v_o}{v_i} = A_F \left(\frac{j2\pi fRC}{1 + j2\pi fRC}\right)$$

$$\Rightarrow \boxed{\frac{v_o}{v_i} = A_F \left[\frac{j(f/f_c)}{1 + j(f/f_c)}\right]}$$

where $A_F = \left(1 + \frac{R_f}{R_1}\right)$ = Pass-band Gain of the filter

f = frequency of the i/p signal

$f_c = \frac{1}{2\pi RC}$ cutoff freq. of the filter (Hz)

The gain-magnitude

$$\left|\frac{V_0}{V_i}\right| = \frac{A_F (f/f_c)}{\sqrt{1 + (f/f_c)^2}}$$

phase angle,

$$\phi = 90^\circ - \tan^{-1}(f/f_c) = 90^\circ - \tan^{-1}(\omega RC)$$

Operation of the Filter

1> At very low frequencies, i.e $f < f_c$

$$\left|\frac{V_0}{V_i}\right| < A_F$$

2> At $f = f_c$, $\left|\frac{V_0}{V_i}\right| = \frac{A_F}{\sqrt{2}} = 0.707 A_F = -3dB$, $\phi = 45^\circ$.

3> At $f >> f_c$, $\left|\frac{V_0}{V_i}\right| \cong A_F$

Filter Design

A high-pass active filter can be designed by implementing the following steps :—

1> A value of the cut-off freq., ω_c (or f_c) is chosen.

2> A, value of the capacitance C, usually betn 0.001 & 0.1uF is selected,

3> The value of the resistance R is calculated using the relation,

$$R = \frac{1}{\omega_c C} = \frac{1}{2\pi f_c C}$$

4) Finally, the values of R_1 & R_f are selected depending on the desired pass-band gain, using the relation, $A_F = \left(1 + R_f/R_1\right)$

Band-Pass Active Filter

A band-pass filter has a pass-band between two cut-off frequencies f_{cL} (lower cut-off freq) & f_{cU} (upper cut-off freq) such that $f_{cU} > f_{cL}$. Any input frequency outside this pass-band is attenuated.

Bandwidth

$$BW = (f_{cU} - f_{cL})$$

If f_{cL} & f_{cU} are known, the resonant freq. can be found from

$$f_r = \sqrt{f_{cL} f_{cU}}$$

If 'f_r' & BW are known, cut-off frequencies are found from,

$$f_{cL} = \left(\sqrt{\left(\frac{BW}{2}\right)^2 - f_r^2}\right) - \left(\frac{BW}{2}\right)$$

$$f_{cU} = (f_{cL} + BW)$$

Types

1) **Wide Band Pass filter** :- Wide-band filter has a bandwidth that is two or more times the resonant frequency i.e $Q \leq 0.5$.
It is made by cascading a low-pass & a high-pass filter circuit.

2) **Narrow Band Pass filter** - A narrow band filter has a quality factor, $Q > 0.5$
It is made by using a single op-amp and multiple feed back circuits.

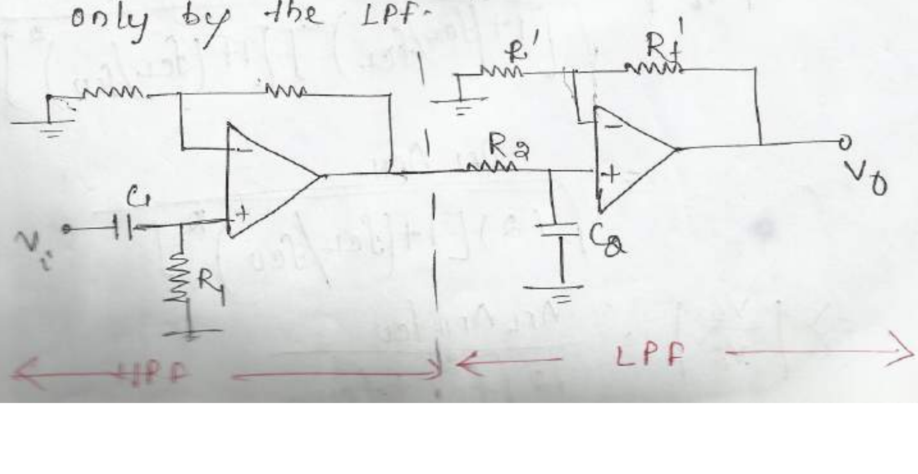
1) Wide Band Pass Active Filter

characteristics

(i) The cut-off frequency of LPF should be 10 or more times the cut-off frequency of the HPF.

(ii) The lower cut-off frequency, f_{CL}, will be determined only by the HPF.

(iii) The higher cut-off freq., f_{CH}, will be determined only by the LPF.



Here $f_{CL} = \frac{1}{2\pi R_1 C_1}$, $f_{CU} = \frac{1}{2\pi R_2 C_2}$

The voltage gain magnitude of the BPF is equal to the product of the voltage gain magnitudes of the HPF & the LPF.

$$\left|\frac{V_0}{V_i}\right| = \frac{A_{FL} A_{FH} (f/f_{CL})}{\sqrt{1+(f/f_{CL})^2}\sqrt{1+(f/f_{CU})^2}}$$

where, A_{FL}, A_{FH} = Pass-band gain of LPF & HPF

f = frequency of input signal (Hz)

f_{CL} = lower cut-off frequency (Hz)

f_{CU} = upper cut-off frequency (Hz)

Cases.

(i) At the centre freq, $f_C = \sqrt{f_{CL} f_{CU}}$

the Gain is, $\left|\frac{V_0}{V_i}\right| = K = A_{FL} A_{FH} \frac{f_{CU}}{f_{CL}+f_{CU}}$

(ii) At $f = f_{CL}$, $\left|\frac{V_0}{V_i}\right| = \frac{A_{FL} A_{FH} (f_{CL}/f_{CL})}{\sqrt{\left[1+(f_{CL}/f_{CL})^2\right]\left[1+(f_{CL}/f_{CU})^2\right]}}$

$$= \frac{A_{FL} A_{FH}}{\sqrt{(2)\left[1+(f_{CL}/f_{CU})^2\right]}}$$

$$\Rightarrow \left|\frac{V_0}{V_i}\right| = \frac{A_{FL} A_{FH} f_{CU}}{\sqrt{2}\sqrt{f_{CL}^2 + f_{CU}^2}}$$

(iii) $f = f_{CU}$, $$\left|\frac{v_0}{v_i}\right| = \frac{A_{FL} A_{FH} \left(f_{CU}/f_{CL}\right)}{\sqrt{(2)\left[1+\left(f_{CU}/f_{CL}\right)^2\right]}}.$$

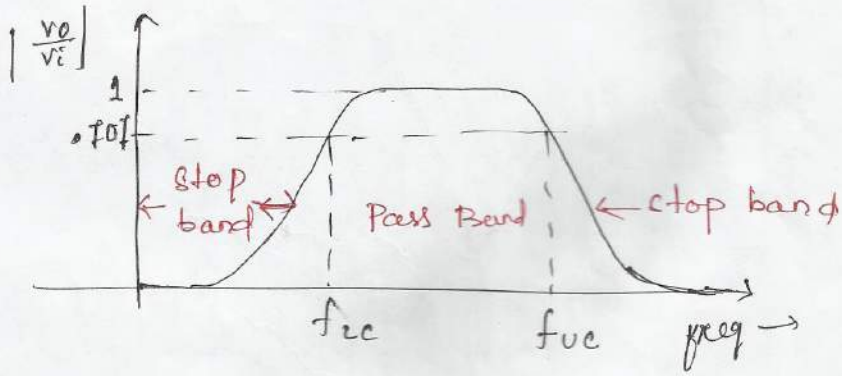
$$\Rightarrow \left|\frac{v_0}{v_i}\right| = \frac{A_{FL} A_{FH} f_{CL}}{\sqrt{2}\sqrt{f_{CL}^2 + f_{CU}^2}}$$

(iv) At $f = f_{CL} = f_{CU}$

$$\text{Gain} \left|\frac{v_0}{v_i}\right| = \frac{A_{FL} A_{FH}}{2}$$

~~Band - Reject (Notch) Active Filter~~

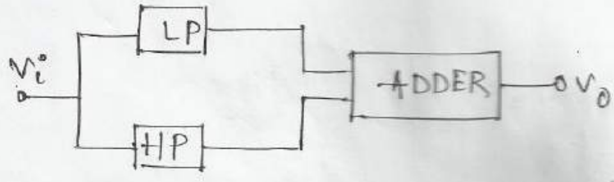
Frequency Response

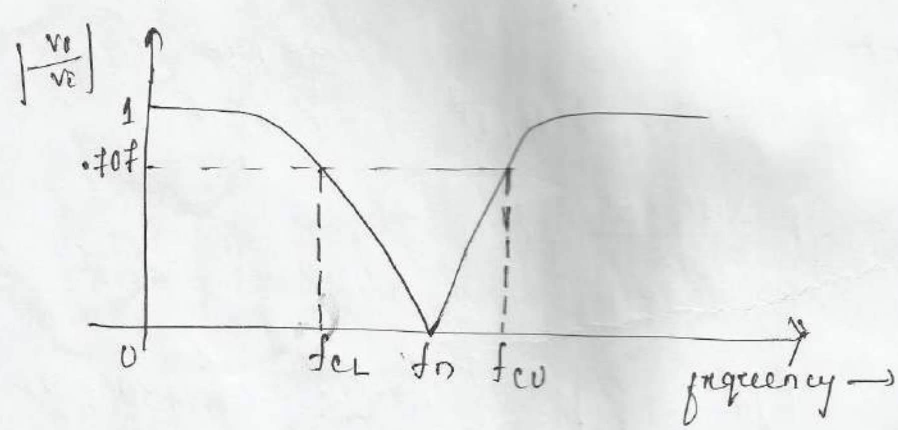


Band-Reject (Notch) Active Filter

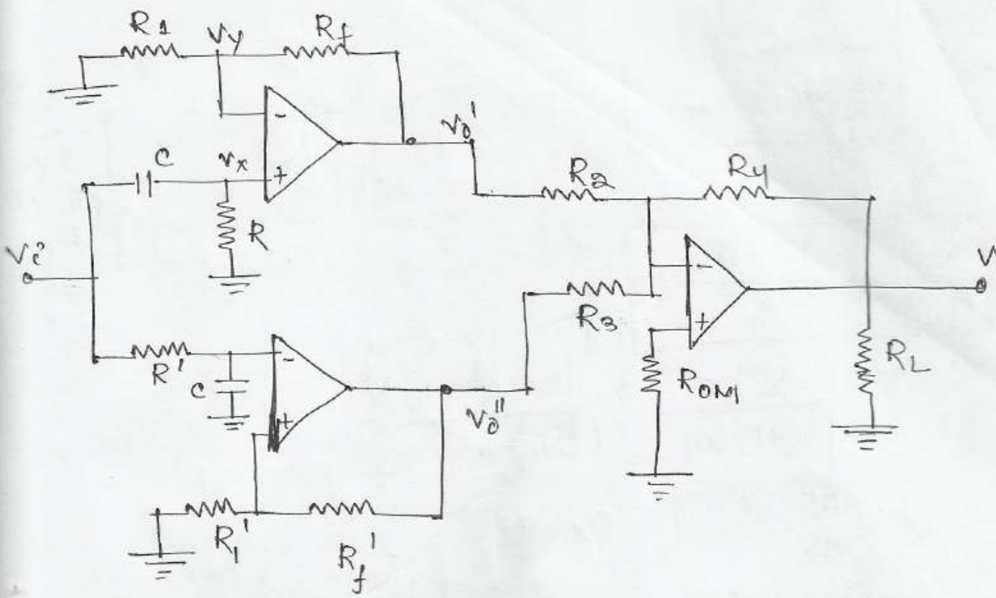
It may be obtained by the parallel connection of a high-pass section with a low-pass section. The cut-off frequency of the high pass section must be greater than that of the low-pass section.

The outputs of HP & LP sections are fed to an adder whose output voltage V_0 will have the notch filter characteristics.





(frequency response)



$$V_0 = A_{FH}\left[\frac{j(f/f_{CH})}{1+j(f/f_{CH})}\right] + A_{FL}\left[\frac{1}{1+j(f/f_{CL})}\right]$$