BIJU PATNAIK UNIVERSITY OF TECHNOLOGY, ODISHA

Lecture Notes

On

QUANTITATIVE TECHNIQUE-II

SIMULATION

Compiled by,

Dr. Subhendu Kumar Rath,

BPUT, Odisha.

<u>**QUANTITATIVE TECHNIQUE-II**</u>

What is Simulation?
A simulation is an imitation of some real thing, state of affairs, or process.
The act of simulating something generally entails representing certain key characteristics or behaviors of a selected physical or abstract system.

Advantages of Simulation
(1)Most complex, real-world systems with stochastic elements cannot be accurately described by a mathematical model that can be evaluated analytically. Thus, a simulation is often the only type of investigation possible.
(2)Simulation allows one to estimate the performance of an existing system under some projected set of operating conditions.
(3)Alternative proposed system designs (or alternative operating policies for a single system) can be compared via simulation to see which best meets a specified requirement.
(4)In a simulation we can maintain much better control over experimental conditions than would generally be possible when experimenting with the system itself.
(5)Simulation allows us to study a system with a long time frame.
e.g., an economic system in compressed time, or alternatively to study the detailed workings of a system in expanded time.

Disadvantages of Simulation
(1)Each run of a stochastic simulation model produces only estimates of a model's true characteristics for a particular set of input parameters. If a "valid" analytic model is available or can be easily being developed, it will generally be preferable to a simulation model.
(2)Simulation models are often expensive and time consuming to develop.
(3)If a model is not a "valid" representation of a system under study, the simulation results, no matter how impressive they appear, will provide little useful information about the actual system.
(4)In some studies both simulation and analytic models might be useful. In particular, simulation can be used to check the validity of assumptions needed in an analytic mode.
(5)On the other hand, an analytic model can suggest reasonable alternatives to investigate in a simulation study.

Differentiate between discrete-event simulation and continuous simulation.
Ans.-: A discrete-event simulation is one where changes in the state of the system occur instantaneously at random points in time as a result of the occurrences of discrete events. For example, in a queuing system where the state of the system is the no of customers in the system, the discrete events that change this state are the arrival of a customer and the departure of a customer due to the completion of its service.

A continuous simulation is one where changes in the state of the system occur continuously over time. For example, if the system of interest is an airplane in flight and its state is defined as the current position of the airplane, then the state is changing continuously over time.

Q.) Consider the Markov chain that has the following (one-step) transition matrix.

State 0 1 2 3 4

$$P = \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{bmatrix} 0 & 0.2 & 0.5 & 0.3 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0.2 & 0 & 0.1 & 0.7 \\ 0 & 1 & 0 & 0 & 0 \\ 0.8 & 0.1 & 0 & 0.1 & 0 \end{bmatrix}$$

(a) Determine the classes of this Markov chain and, for each class, determine whether it is recurrent or transient.

States 1 and 3 are accessible from each other (p_{31} = 1 and p_{13} = 1), but no other states are accessible from these states (p_{1j} = 0 and p_{3j} = 0 for j = 0, 2, 4). Therefore, states 1 and 3 communicate and form one class of the Markov chain. Upon entering either state, the process will return to that state in two steps, so {1, 3} is a recurrent class.

State 0 is accessible from state 4 (p_{40} = 0.8), state 2 is accessible from state 0 (p_{02} = 0.5), and state 4 is accessible from state 2 (p_{24} = 0.7), so each of these states is accessible from each of these other states. Therefore, states 0, 2, and 4 communicate and form a second class of the Markov chain. The process can move from any of these states to state 1 or state 3, in which case the process never would return to states 0, 2, and 4 again. Therefore, {0, 2, 4} is a transient class.

(b) For each of the classes identified in part (a), determine the period of the states in that class.

We calculate $P^{(2)}$ and $P^{(3)}$.

$$P^{(2)} = P \times P = \begin{bmatrix} 0 & 0.4 & 0 & 0.25 & 0.35 \\ 0 & 1 & 0 & 0 & 0 \\ 0.56 & 0.17 & 0 & 0.27 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0.26 & 0.4 & 0.34 & 0 \end{bmatrix}$$

$$P^{(3)} = P^{(2)} \times P = \begin{bmatrix} 0.28 & 0.285 & 0 & 0.435 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0.382 & 0.28 & 0.338 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 0.3 & 0.28 \end{bmatrix}$$

Since $p_{11} = p_{33} = 0$ and $p^{(2)}_{11} = p^{(2)}_{33} = 1$, the class {1, 3} has period 2.

Now note that $p_{00} = p_{22} = p_{44} = 0$, $p^{(2)}_{00} = p^{(2)}_{22} = p^{(2)}_{44} = 0$, and $p^{(3)}_{00} > 0$, $p^{(3)}_{22} > 0$, $p^{(3)}_{44} > 0$. This indicates that the class {0, 2, 4} has period 3.

MCA(Model Set 1)
Fifth Semester Examination-2012
QUANTITATIVE TECHNIQUES-II
Full Marks-70
Time-3 Hours

Answer Question No. **1** which is compulsory and any **five** from the rest.

1. Answer the following questions: [2×10]
   (a)What is a stochastic process? What are the types of stochastic process?
   (b)Define transition probabilities and stationary transition probabilities.
   (c)Define transient, recurrent (persistent) and absorbing states.
   (d)What do you mean by steady state probabilities? Write down the steady state equations.
   (e)Define a continuous time Markov Chain.
   (f)What is Markov Decision Process?
   (g)Define stationary policy, deterministic policy and randomized policy.
   (h)What is a pseudo random number? Write down two methods for generating pseudo random number.
   (i)What are the key elements of discrete-event simulation? Write down those elements.
   (j)What do you mean by antithetic variable and control variate.
   (k)What is stratified sampling estimator?
2. (a)Classify each state of the Markov Chain i.e., whether recurrent, transient or absorbing state. Also find period of each state. [5]

$$\begin{bmatrix} 0 & 1/3 & 1/3 & 1/3 \\ 1/3 & 0 & 1/3 & 1/3 \\ 1/3 & 1/3 & 0 & 1/3 \\ 1/3 & 1/3 & 1/3 & 0 \end{bmatrix}$$

   (b)State and prove Chapman-Kolmogorov equations. [5]
3. A production process contains a machine that deteriorates rapidly in both quality and output under heavy usage,so that it is inspected at the end of each day. Immediately after inspection, the condition of the machine noted and classified into one of four possible states:

States	Condition
0	good as new

1 operable-minor deterioration
2 operable-major deterioration
3 inoperable and replaced by a good as new machine

The process can be modeled as a Markov Chain with its (one-step) transition matrix P given by

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 7/8 & 1/16 & 1/16 \\ 0 & 3/4 & 1/8 & 1/8 \\ 0 & 0 & 1/2 & 1/2 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(a) Find the steady state probabilities.
(b) If the costs of being in states 0,1,2,3 are \$0, \$1,000, \$3,000 and \$6,000 respectively, what is the long run expected average cost per day?
(c) Find expected recurrence time for state 0 i.e., μ_{00}. [10]

4. (a)What do you mean by absorption probability? Where it is used? Find the probability of absorption from state 2 to state 0 i.e., f_{20}.

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 2/3 & 0 & 1/3 & 0 & 0 \\ 0 & 2/3 & 0 & 1/3 & 0 \\ 0 & 0 & 2/3 & 0 & 1/3 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$ [5]

(b)The state of a particular continuous time M.C is defined as the no of jobs currently at a certain work center, where a maximum of three jobs are allowed. Jobs arrive individually whenever fewer than three jobs are present, the time until next arrival has an exponential distribution with a mean of ½ day. Jobs are processed at the work center one at a time and then leave immediately. Processing times have an exponential distribution with mean of ¼ day.

(i) Constuct the rate diagram for this M.C.
(ii) Write the steady state equations.
(iii) Solve these equations for the steady state probabilities. [5]

5. (a)Formulate the Markov Decision Process as an LPP. [5]
(b)Evaluate the following integral using Monte Carlo approach

$$\int_0^1 \int_0^1 e^{x+y} dxdy$$ [5]

6. (a)Using proper transformation evaluate the integral

$$\int_{-2}^{2} e^{x+x^2} dx$$ [5]

(b)Find out the recurrence relationship for a Poisson random variable X as P{X=i+1}=[λ/(i+1)]P{X=i} and then write the algorithm to generate X using random numbers. [5]

7. (a)Generate a standard normal variate Z with mean 0 and variance 1 having density

   $f(x) = \frac{2}{\sqrt{2\pi}} e^{-x^2/2}$,0<x<∞ using rejection method. [5]

   (b)Define a Poisson process.Generate the first T time units of a non-homogeneous Poisson process. [5]
8. (a)Describe a queuing system with two servers in series. [5]

   (b)Differentiate between Chi-square and Kolmogorov-Smirnov test. [5]
9. (a)Suppose that over a 30-day trial period there are 6 days in which no accidents occurred,2 in which 1 accident occurred,1 in which 2 accidents occurred,9 in which 3 occurred,7 in which 4 occurred,4 in which 5 occurred and 1 in which 8 occurred,Test whether these data are consistent with the underlying hypothesis of Poisson distribution. [5]

   (b)Describe the Kolmogorov-Smirnov test for continuous data. [5]

MCA(Model Set 2)
Fifth Semester Examination-2012
QUANTITATIVE TECHNIQUES-II
Full Marks-70
Time-3 Hours

Answer Question No. **1** which is compulsory and any **five** from the rest.

1. Answer the following questions: [2×10]
   (a)What is Markovian Property? Define Markov Chain?
   (b)What are the properties of a tpm (transition probability matrix)?
   (c)Define ergodic state and ergodic Markov chain.Define irreducible MC.
   (d)Define transition intensities in continuous time MC.
   (e)What do you mean by first passage time?
   (f)Define accessible state, communicate state and period of a MC.
   (g)What is optimal maintenance policy?
   (h)Describe a model for Markov Decision Process.
   (i)How will you choose the constants a and m in conguential method of generating pseudo random nos.
   (j)What is importance sampling?
2. (a)Classify the states of the MC as transient, persistent or absorbing state. Find period of each state.

$$P = \begin{bmatrix} 0 & 4/5 & 0 & 1/5 & 0 \\ 1/4 & 0 & 1/2 & 1/4 & 0 \\ 0 & 1/2 & 0 & 1/10 & 2/5 \\ 0 & 0 & 0 & 1 & 0 \\ 1/3 & 0 & 1/3 & 1/3 & 0 \end{bmatrix}$$ [5]

   (b)What is simulation? What are the advantages and disadvantages of simulation? Differentiate between discrete-event simulation and continuous simulation.
   [1+2+2]
3. A production process contains a machine that deteriorates rapidly in both quality and output under heavy usage,so that it is inspected at the end of each day. Immediately after inspection, the condition of the machine noted and classified into one of four possible states:

States	Condition
0	good as new
1	operable-minor deterioration

2	operable-major deterioration
3	inoperable and replaced by a good as new machine

The process can be modeled as a Markov Chain with its (one-step) transition matrix P given by

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 7/8 & 1/16 & 1/16 \\ 0 & 3/4 & 1/8 & 1/8 \\ 0 & 0 & 1/2 & 1/2 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Decisions taken and the relevant states are given as follows:

Decision	Action	Relevant states
1	do nothing	0,1,2
2	overhaul	2
3	replace	1,2,3

Whenever the machine is overhauled a maintenance cost of $2,000 is incurred as well as $2,000 is lost due to unavailable of that machine for production during maintenance.When the machine is replaced $4,000 is incurred for buying the new machine and lost profit is $2,000 because of unavailability during that period.

Design various policies and find optimal policy by finding out their expected average cost. [10]

4. (a)If x_0=3 and $x_n=(5x_{n-1}+7) \bmod 200$, find $x_1 \ldots x_{10}$. [5]
   (b)Estimate the value of π using random numbers. [5]
5. (a)Give an efficient algorithm to simulate the value of a random variable X such that P{X=1}=0.3,P{X=2}=0.2,P{X=3}=0.35,P{X=4}=0.15 [5]
   (b)Find out the recurrence relation for the binomial(n,p) random variable X such that P{X=i+1}=[(n-i)/(i+1)][p/(1-p)]P{X=i} and then write down the algorithm to generate X using random nos. [5]
6. (a)Generate the value of an exponential random variable X using inverse transform method. [5]
   (b)Use rejection method to generate a random variable having density function $f(x)=20x(1-x)^3, 0<x<1$. [5]
7. (a)Describe the single server queuing system. [5]
   (b)Describe the polar method for generating normal random variables. [5]
8. (a)Explain how antithetic variables can be used in obtaining a simulation estimate of the quantity

$$\theta = E(U) = \int_0^1 e^x dx \,.$$

Calculate the variance reduction of using antithetic vaiable over the use of independent random variables. [5]

(b)Use control variate to estimate θ as in Qn.8(a) and find the variance reduction. [5]